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J M Keynes's mathematical style: Very concise, precise, and exact

Michael Emmett Brady[1*]

[1]California State University, Dominguez Hills, 1000 E. Victoria Street, Carson, CA, Carson, CA 90747, United States. E-mail: mandmbrady@juno.com

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Abstract

Keynes's mathematical style, starting with his First Fellowship Dissertation in 1907 for Cambridge University, England, through his formulation of a linear, first order difference equation that incorporated the interaction of the Multiplier and Accelerator (called the Relation) for Harrod's use in his August,1938 correspondence with Harrod, and ending with his exchanges over probability and statistics with J. Tinbergen, an advocate of the Limiting Frequency Interpretation of Probability in 1939-40, was always very concise, precise and exact. Specifically, Keynes always provided the first steps in his mathematical analysis and the last step. However, he would rarely put in the intermediate steps. Keynes's view was that he always provided a clear, literary, prose explanation of his analysis that would allow any reader of his work to grasp the same basic, fundamental points that were being made in the mathematical analysis. A reader concentrating on Keynes's supplementary mathematical analysis would also grasp the basic fundamental point being made. The intermediate mathematical steps in a Keynesian analysis need to be formulated by working backwards from the final step by the reader. This is precisely what economists have failed to do. They have been unable to generate the intermediate steps that connect the first and last steps. For instance, the results that Keynes presented in the *General Theory* regarding his IS-LM (LP) and D-Z models were never correctly grasped by any mainstream economist in the 20th and 21st centuries because they were not able to reconstruct the mathematical analysis in chapters 20 and 21 of the *General Theory*. All current contributions to macroeconomic history and history of economic thought state that, at best, Keynes only had an intuitive, implicit understanding of the multiplier before 1931. Supposedly, Kahn used mathematical and logical analysis to derive the multiplier in his June, 1931 *Economic Journal* (EJ) article. He then taught Keynes the technical steps from his article. Therefore, it would have been impossible for Keynes to write the *General Theory* without Kahn's contribution and instruction. This paper shows that it was Keynes, not Kahn, who developed the mathematical and logical analysis of the multiplier in his A Treatise on Probability ten years before Kahn's article was published.

Keywords: *Probability, Expected value, Risk, Multiplier*

1. Introduction

This paper will concentrate on illustrating Keynes's style by analyzing the mathematical connection between Keynes's mathematical results in chapter 26 on page 315 in footnote one of the *A Treatise on Probability* and the identical type of result that appears on page 183 of Richard Kahn's June, 1931 article in the *Economic Journal*. It will be demonstrated

* *Corresponding author: Michael Emmett Brady, California State University, Dominguez Hills, 1000 E. Victoria Street, Carson, CA, Carson, CA 90747, United States. E-mail: mandmbrady@juno.com*

that, once the differences in mathematical notation are compared and taken into account, the results presented by Kahn are identical to those presented by Keynes in 1921 in the *A Treatise on Probability* and reflect a style of presentation that is identical to Keynes's mathematical style.

We will also examine the belief among modern economists that Keynes's *General Theory* does not use math except for defining or conceptualizing relationships, in the form of $y = f(x)$ or $z = g(q)$ type notation, like the $D = \Phi(N)$ and $Z = \theta(N)$ specifications that appear in chapter 3 of the *General Theory*. Such economists (Hayes, 2007) will be completely lost when they reach chapters 20 and 21 of the *General Theory*, where Keynes derives and presents his actual mathematical models (Brady, 2004; Brady, 2011; Arthmar and Brady, 2007).

This paper overturns all existing work done in macroeconomic history and history of economic thought regarding the origins and development of the multiplier used in the GT. All existing academic work assigns credit for the mathematical and logical development of the multiplier only to R. Kahn as presented in his *EJ* article of June ,1931.The aim of this paper is to show that Keynes had already developed the mathematical and logical theory of the multiplier in 1921 in his *A Treatise on Probability*(TP).This is accomplished by analyzing a mathematical contribution provided by Keynes in chapter 26 on page 315 in footnote 1 of the *TP*.

The paper was formulated in five sections. Introduction was given in section one.. Section two examined the mathematical exposition in Keynes's footnote one on page 315 of the *A Treatise on Probability* (TP, 1921) and show what he had left out because he expected a reader to be able to follow up and put in the missing steps. Even a mathematical genius like Paul Samuelson overlooked what Keynes was doing in this footnote. Section three looks at Kahn's mathematical analysis contained on page 183 in his June 1931 *Economic Journal* article and demonstrates that Kahn's style of presenting mathematical results in this article is identical to Keynes's style of presentation, where no intermediate results are presented.

Section four looks at the assessments of Keynes's mathematical style based on the analysis of R. Backhouse and B. Bateman, who are very representative of economists in general, as well as the assessments of Pierre Lamieux, Robert Cord, and Robert Skidelsky regarding Keynes's view of mathematics. The beliefs of R. Backhouse and B. Bateman, that Keynes's footnotes in the *General Theory* (GT,1936) are minor and insignificant, is due to the inability of Backhouse and Bateman to grasp the fact that none of the intermediate steps are included in any of the footnotes in chapters 10, 20, and 21 of the GT. The inclusion of the intermediate steps adds at least 25 pages to the length of the GT. Once these missing steps are included, it is clear that the GT is, for its time**,** a heavily, mathematical exposition, similar in degree to Pigou's The Theory of Unemployment (TTOU, 1933), but which was not based on the Marshallian, partial equilibrium, ceteris paribus approach of Marshallian economics. Section five concludes the paper.

2. Keynes's approach to mathematics in the TP

Keynes presents the following technical analysis in his footnote. He expects the reader to be able to deploy correctly that part of the differential calculus that deals with taking the limits of different kinds of series. Economists in the 1930s, such as R. Hawtrey, D. Robertson, and J. Robinson, who either lacked knowledge of calculus or had forgotten how to apply it, would not be able to grasp what Keynes was doing. Keynes addressed these economists on pp. 122-123 of the GT to point out to them that there was a difference between the concept of logical time deployed mathematically in the theory of the multiplier using differential calculus and actual, historical time, which, of course, can't possibly be instantaneous, but will take time for the process to work its way out.

2.1. Consider Keynes's analysis on page 315 of the TP

The 'risk' may be defined in some such way as follows. If A is the amount of good which may result, p its probability ($p + q = 1$), and E the value of the 'mathematical expectation,' so that $E = pA$, then the 'risk' is R, where $R = p(A - E) = p(1 - p)A = pqA = qE$. This may be put in another way: E measures the net immediate sacrifice which should be made in the hope of obtaining A; q is the probability that this sacrifice will be made in vain; so that qE is the 'risk.'*

$E + R_1 + R_2 + \ldots = E(1 + q + q^2 + \ldots) = E/(1 - q) = E/p = A$." (Keynes, 1921, p. 315).

where,

E stands for expected value or expectation .E is defined as the probability of an outcome times the outcome. Given that p is the probability of success, q is the probability of failure, p and q are additive so that p+q=1, and A is the outcome, then E=pA.

Note: * Keynes's actual analysis as presented in ft. 1 on p.315 of his TP, 1921.

R is defined as Risk, where R equals the probability, q, of **not** obtaining the expected value, which equals pA, times pA. Therefore, q(pA) =qE is the probability of **not** obtaining the expected value.

Keynes has presented the first two steps and then his final result. He does that same thing in his correspondence with Harrod in August 1938, where Keynes presented the first technical analysis of the interaction of the multiplier and accelerator in a discussion of dynamic, intertemporal growth.

Keynes expects a mathematically literate reader of this footnote to recognize that, because p and q $(1 - p)$ are fractions, the terms q^1, q^2, q^3, etc. are a declining series of numbers. The … dots mean that it is an infinite series. The problem involves variables raised to increasingly higher exponents or powers, so that it is a geometric series and could not be an arithmetic series.

Keynes's term," $= E/(1 - q)$ " is the final step in the mathematical process of taking a limit to sum the series of numbers. Keynes has taken the limit of a declining, geometric, infinite series to obtain $E/(1-q)$ or $E/p.1/(1-q)$ or $1/p$ is the multiplier. The multiplicand is E. Keynes's GT Investment Multiplier is mathematically, logically, and technically identical to the TP Multiplier except in terms of the notation used.

Paul Samuelson missed a golden opportunity to write the final chapter of the history of the multiplier in 1977 in his article in the *Journal of Economic Literature.* One of the sections in his 1977 paper dealing with the St. Petersburgh paradox considered Keynes's risk model, R, from chapter 26 of the TP. Samuelson overlooked that this exposition is a special case of Keynes's general model presented on pp.353-359 of the 1921 TP. Samuelson needed to have studied page 315 of the TP, when examining Keynes's simplified risk model because the simplified model appears only on this page of the TP. Samuelson was the only economist to take Keynes's R model seriously. Unfortunately, Samuelson overlooked the first footnote at the bottom on page 315, where Keynes demonstrated that the process of taking the limit of a geometrical, declining, infinite series led to a finite answer. Keynes‘s answer in the footnote on page 315 of the TP is identical to the answer given in the *General Theory,* once it is recognized that $p + q = 1$, where p is the probability of success and q is the probability of failure, is replaced by $mpc + mps = 1$, where mpc is the marginal propensity to consume and mps is the marginal propensity to save.

3. Kahn's mathematically identical exposition in June, 1931 in the *Economic Journal* on p. 183

Kahn's actual analysis as presented on p.183 of his June, 1931 EJ article.
"Let the employment of each additional man involve a net increase in the rate of expenditure on home-produced consumption—goods of mW out of his wages and of nP out of the addition to profits with which his employment is associated. Then the total increase in the rate of expenditure on home-produced consumption- goods are:

$mW + nP.$

The direct result is a further addition to the volume of employment [2] of amount

It follows that for each man placed in primary employment, the number who receive secondary employment is:

$$k + k^2 + k^3 + \ldots = k / (1 - k) \qquad \ldots(1)$$

and the ratio of secondary employment to primary employment is $k/1 - k$. " (Kahn, 1931, p. 183).

There is no supporting, technical analysis provided by Kahn to support any part of his presentation on p.183 of his *EJ* article. Kahn does not explain how he derives the finite sum that leads to a finite numerical answer. He never specifies that he is dealing with a geometrical, declining, infinite series of numbers. The concept of a limit is never discussed at any time by Kahn anywhere in the article. He never cites any mathematical treatise dealing with limits or infinite series of numbers. He just presents the final answer after specifying the initial step, which is identical to Keynes's general style of presentation.

In fact, Kahn's entire presentation style in his June 1931, EJ article on p.183 is identical to Keynes's presentation style. All one needs to do to obtain the same form, as presented by Keynes in 1921 in the TP, is to factor out the k to obtain

$k\,(1 + k + k^2 + k^3 \ldots) = k \bullet [1 / (1 - k)]$. This is mathematically identical to Keynes

$E\,(1 + q + q^2 + \ldots) = E / (1 - q) = E \bullet [1/(1 - q)]$ result from p. 315 of the TP.

Kent provides the following footnote 19 at the end of his article in 2007 in *HOPE*:

"In May 1938, in a letter to Colin Clark concerning his article "The Determination of the Multiplier from National Income Statistics" that was published in the September 1938 *Economic Journal*, Keynes has a section titled "History of the Multiplier Doctrine," in which he said:

> One must distinguish here between some sort of formal statements such as was given in Kahn's *EJ* article and the general notion of there being such a thing as secondary employment.

If one is to include unpublished memoranda then it must be remembered that the original draft of Kahn's theory was contained in a memorandum which he wrote (as you will remember) for the Economic Advisory Council in the late summer of 1930. . . The general notion of secondary employment, however, must go back much further. For example, it is clearly explained in *Can Lloyd George do it?* by Hubert Henderson and myself, where we used the argument that because of secondary employment the aggregate saving on the dole would pay half the capital cost of public works. (Keynes, 1983b).

So, in discussing the history of the multiplier, Keynes himself does not refer to his speech on the hustings in May 1929."(Kent, 2007).

Keynes considered his May 1929 application of the logical theory of the multiplier to be a part of his reference to

"*Can Lloyd George do it?* by Hubert Henderson and myself, where we used the argument that because of secondary employment the aggregate saving on the dole would pay half the capital cost of public works" (Kent, 2007).

Why didn't Keynes mention his footnote on page 315 of the TP? The answer is that doing this was not part of his mathematical style of presentation. He assumed that a reader of the GT's chapter 10, who was interested in the details of the logical theory of the multiplier, would have the mathematical training to work it out for himself. Similar examples, for instance, of Keynes's approach are Keynes's derivation of a first-order, difference equation, in his correspondence with Harrod in August of 1938 on an article submitted by Harrod on economic growth to the *EJ*, demonstrating the interaction of the multiplier and the accelerator, and Keynes's letter of August 27th,1935 to Harrod, demonstrating to Harrod that the classical theory of the rate of interest amounted to a single, downward sloping curve in (Y,r) space that could not possibly serve to determine an equilibrium, determinate rate of interest. Harrod immediately recognized what Keynes had done, as he also had recognized in his reply to Keynes in 1938. He responded in his letter of August 30th, 1935 that Keynes had effected a "radical reconstruction" of the theory of the rate of interest and acknowledged that Keynes had provided the missing, mathematical equation, the Liquidity Preference Function equation of page 199 of the GT, that had to be specified in [Y(income), r(rate of interest)] space so that there could be an intersection of two curves that would result in a determinate, equilibrium position for the rate of interest based on a set of simultaneous equations. Of course, the result is Keynes's IS-LM (LP) model as specified on pp.298-299 of the GT.

4. The failure to comprehend Keynes' mathematical style leads only to error

Consider the following assessment of the role of mathematics in the GT made by the French economist, Pierre Le Mieux:

"Many urban legends circulate about the use of mathematics in economics.

For example, many people think (they have heard that…they know somebody who knew somebody who knew…) that John Maynard Keynes loved and used mathematics. In his 1936 magnum opus, *The General Theory of Employment, Interest, and Money*, Keynes seldom used math. He even attacked economists for using them too much. One can find this by simply reading the *General Theory*:

> It is a great fault of symbolic pseudo-mathematical methods of formalizing a system of economic analysis…that they expressly assume strict independence between the factors involved and lose all their cogency and authority if this hypothesis is disallowed; whereas, in ordinary discourse, where we are not blindly manipulating but know all the time what we are doing and what the words mean, we can keep "at the back of our heads" the necessary reserves and qualifications and the adjustments which we shall have to make later on, in a way in which we cannot keep partial differentials "at the back" of several pages of algebra which assume that they all vanish. Too large a proportion of recent "mathematical" economics are mere concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols.

Interestingly, Paul Samuelson, who did use mathematics a lot (he much contributed to formalizing economic theory with his 1947 *Foundations of Economic Analysis*), was not as uncritical as many think. One must read his 1952 *American Economic Review* article "Economic Theory and Mathematics –- An Appraisal". "Any truth arrived at by way of mathematical manipulation must be translatable into words", he wrote; "and hence, as a matter of logic, could quite possibly have been arrived at by words alone" (Le Mieux, 2014).

Lemieux's erroneous belief, that "In his 1936 magnum opus, *The General Theory of Employment, Interest, and Money*, Keynes seldom used math.", results from his failure to examine the many footnotes in the GT in chapters 10, 20, and 21. It is simply impossible to understand how Keynes derives his IS-LM (LP) and D-Z models in chapters 20 and 21 of the GT without a careful examination of the footnotes and mathematical analysis contained in chapters 20 and 21.

Consider La Mieux's further quotation from the GT:

"He even attacked economists for using them too much. One can find this by simply reading the *General Theory*:

It is a great fault of symbolic pseudo-mathematical methods of formalizing a system of economic analysis…that they expressly assume strict independence between the factors involved and lose all their cogency and authority if this hypothesis is disallowed; whereas, in ordinary discourse, where we are not blindly manipulating but know all the time what we are doing and what the words mean, we can keep "at the back of our heads" the necessary reserves and qualifications and the adjustments which we shall have to make later on, in a way in which we cannot keep partial differentials "at the back" of several pages of algebra which assume that they all vanish. Too large a proportion of recent "mathematical" economics are mere concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols.

Pace Le Mieux, Keynes's attack in the GT is on the Marshallian concentration on a mathematical analysis that is limited to functions with only one independent variable and one dependent variable. Lamieux overlooks that Keynes's refers to this type of mathematics, which overlooks the "…the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols." as "pseudo- mathematical" and "mathematical" economics".

Note that La Mieux is correct about Samuelson's point concerning the use of mathematical analysis in economics and that it is similar to Keynes's views.

Now consider the following assessment made by R. Cord of Keynes's view of applying mathematics in the GT:

"Although Keynes was, of course, the key figure in the development of the ideas that culminated in the *General Theory*, we know that he was able to draw on some crucial help along the way. One of the more obvious examples of this was Kahn's June 1931 article on the multiplier, which provided one of the missing links in the Keynesian system. Even if the debate continues over the origins of the multiplier concept—names mentioned as early contributors include De Lissa, Giblin, Hawtrey, Schwoner, Warming, Wulff, and Keynes himself (see Kent, 2007)—it is difficult to disagree with G. L. S. Shackle's (1951) observation that Kahn's article was "one of the great landmarks of economics." Granted, Keynes and other candidate precursors clearly recognized the importance of the multiplier, but it was Kahn who was the first to bring together in a rigorous, detailed, and structured way the mechanisms by which the multiplier process works. Cambridge's ownership of the concept was bolstered by the multiplier analyses carried out by Keynes (Kent, 2005) and Colin Clark (1938).

Despite Keynes's aversion to applying mathematics to economics, he did set some store by the mathematical simplification, where possible, of economic processes. The multiplier was the best example of this and related to it was the consumption function, which Keynes was able to formulate as a result of Kahn's article and which has received extensive subsequent treatment within the $Y = C + I + G$ rubric. (Cord, 2011).

I have already demonstrated in the previous two sections of this paper that Keynes did not draw on Kahn for help in formulating the mathematical theory of the multiplier, that Kahn was not the first "… to bring together in a rigorous, detailed, and structured way the mechanisms by which the multiplier process works", and that Keynes did not need Kahn's article to help him formulate the theory of the multiplier.

Cord's misbelief, that Keynes had an "…aversion to applying mathematics to economics…", is the result of Cord's failure to grasp Keynes's mathematical style, which involved a very concise, precise, and exact manner of presenting mathematical results. Incorporating the intermediate steps would have required the addition of 25 pages to the GT if all of the intermediate steps in chapters 10, 20, and 21 were included.

Note the well-known assessment, expressed to Robert Skidelsky by Richard Kahn in private interviews, that Kahn "… recalled Keynes as being a poor mathematician by 1927." (Skidlesky, 1992).

Finally, consider the analysis and assessment of Keynes's mathematical style by Backhouse. Backhouse makes the following claim:

"An important source for understanding Keynes's use of mathematics in the *General Theory* is found in the lectures he gave in 1932-1933 during the book's preparation. The notes taken by the students contain the remark" …Equations are symbolic rather than algebraic."…This is exactly the way that he had used mathematical notation in the Treatise on Probability…in which he had presented mathematical symbols for probability but then gave them meanings that differed from their usual usage" (Backhouse, 2010).

The student(s) who took the notes and Backhouse, who is relying on student notes, are badly confused. This is what Keynes stated in the *A Treatise on Probability* (TP, 1921):

"The hope, which sustained many investigators in the 19th century, of gradually bringing the moral sciences under the sway of mathematical reasoning, steadily recedes—if we mean, as they meant, by mathematics the introduction of precise numerical methods. The old assumptions, that all quantity is numerical and that all quantitative characteristics are additive, can be no longer sustained. Mathematical reasoning now appears as an aid in its symbolic rather than in its numerical character" (Keynes, 1921).

Keynes's mathematical analysis in the TP is based on Boole's nonlinear, non-additive, non-numerical (interval) approach in his 1854 *The Laws of Thought*, which was the first, formal development of a logical theory of probability in history. Keynes, like Boole, used indeterminate and imprecise interval-valued probability analysis that was not numerical and not additive so that exact and precise numerical answers could not be derived. There is no support for Backhouse's claim that

> "This is exactly the way that he had used mathematical notation in the Treatise on Probability…in which he had presented mathematical symbols for probability but then gave them meanings that differed from their usual usage."

Backhouse's earlier claim, that

> "Intuition was not only the basis for Keynes's early theory of probability, founded on the notion that probabilities were elementary properties that could be assessed through intuition…" (Backhouse, 2010) also makes no sense. Interval-valued probability, not intuition, was the foundation and basis for Keynes's theory of probability. Intuition was needed to be able to access the degree of similarity or dissimilarity that existed between propositions.

Relying on student notes to assess Keynes' approach to mathematics in the GT is highly speculative, more so since the formal mathematical D-Z model in the GT was not taught or lectured on by Keynes in his student lectures between 1932 and 1935. The D-Z mathematical model in the GT is a generalization of the mathematical model in Pigou's *The Theory of Unemployment* (TTOU, 1933). Keynes compares that model to his own in the appendix to chapter 19 of the GT. The explanation of why Keynes did not lecture on Pigou's model in his student lectures because it was very difficult, was given by Hicks:

> "One of the main reasons for this situation is undoubted to be found in the fact that Mr. Keynes takes as typical of "Classical economics" the later writings of Professor Pigou, particularly *The Theory of Unemployment*. Now *The Theory of Unemployment* is a fairly new book and an exceedingly difficult book; so that it is safe to say that it has not yet made much impression on the ordinary teaching of economics. To most people its doctrines seem quite as strange and novel as the doctrines of Mr. Keynes himself; so that to be told that he has believed these things himself leaves the ordinary economist quite bewildered" (Hicks, 1937).

Consider the following claim made by Backhouse:

> "Keynes's method is to specify abstract functional relationships that are then used as the framework around which the discussion is centered. The word "function " is used 109 times in the book, always to denote a mathematical function. There is also a curious asymmetry in the way he uses the terms 'function" and "curve" (De Vroey, 2008).

Keynes uses the term "curve" forty-three times (Hicks, 1937). What is significant about this is that it is always used to refer to older theories. When it comes time to expounding his theory, Keynes talks in terms of "functions". He could have used the same terminology to expound both the classical theory and his own, but he did not (Backhouse, 2010).

First, this is not Keynes's method. Second, I have already analyzed Keynes's use of the term "curve" in section 3 above when developing his approach in GT footnote 2 on pp.55-56 and on p.197. There is simply no support for the claims made by Backhouse. Even more, confusing in this regard is his footnote 11, p.140, which relates to Keynes's generalization of Pigou's elasticity analysis in TTOU:

> "The reader also needs to know that D' and D'' are the first and second derivatives of the function." (Backhouse, 2010, p.140, footnote 11; footnote 11 is on p.146).

Backhouse's references to chapters 20 and 21 (Backhouse, 2010, p.140) of the GT demonstrate that Backhouse does not understand how Keynes presents his results, as he has overlooked the fact that Keynes is generalizing Pigou's model on pp. 281-286 and pp. 304-306 of the GT. Backhouse makes no reference to the appendix to chapter 19, where Keynes compares and contrasts his generalized model to Pigou's special model.

Of course, the first and second derivatives are used to specify the shapes of the D and Z functions when specified as curves, as well as the ASC. Backhouse does not understand what Keynes is doing. The rest of the article (pp. 141-146) is spent by Backhouse in demonstrating this conclusion again and again. Backhouse's misunderstanding of the role of functions and curves in Keynes's GT is not isolated. Consider the equally flawed claims made about Keynes's modeling in the GT below by others.

"Finally, a technical note on Keynes's method. Roger Backhouse (2010) has provided an excellent defense of Keynes's method, in particular**,** his reluctance to use mathematical models. (Keynes was a good mathematician, and his Ph.D. thesis—later published—was on probability theory.) The earliest reviews of the GT complained about Keynes's *excessive* use of mathematics in the book—something the modern reader finds surprising because there are few equations (all using simple math) and only one diagram. But this is not simply because economics came to rely so heavily on mathematics. Backhouse shows that the GT is permeated by a mathematical way of thinking—beginning with intuition and clear thinking, with details added later as Keynes constructed incomplete models, explained mostly verbally. Keynes believed there were too many qualifications, reservations, adjustments, and interdependencies that precluded specification in formal mathematics. In other words, to keep his theory *general* he had to keep it somewhat vague. It was precise because postwar "Keynesian" economics translated the GT into algebra that it became too simplistic and specific to be relevant to our complex world. The post-1970s developments further mathematized economics in an attempt to make it even more rigorous. (Wray, 2011).

The errors made by Wray are corrected below:

First, Keynes was only reluctant to use mathematical models in economics if they were based on the assumptions of additivity and linearity, or were Marshallian in nature so that only functions of the type $y = f(x)$ were considered.

Second, Keynes never did a Ph.D. thesis. He did a Fellowship Thesis.

Third, Keynes left out all of the initial, beginning, and intermediate steps in his mathematical analysis in the GT in chapters 10, 20, the appendix to chapter 19 and 21. Putting in all of the missing steps would add 25 pages to the GT.

Fourth, All of Keynes's models are complete but very concise.

Finally, Keynes never made his *General Theory* vague. It is as clear as can be for any mathematically literate reader.

Continuing, Carabelli, and Cedrini make the following argument based on Backhouse (2010):

"Yet, consistently with his preference for 'symbolic' rather than 'algebraic' equations, the latter requiring 'assumptions which are too much simplified' (Rymes, 1989), Keynes used functional relationships and abstract (i.e. not precisely specified) functions only. Moreover, he used the more precise term 'curve' when exposing the classical theory but always opted for the more abstract term 'function' when expounding his theory (Backhouse, 2010). He employed the strong term 'determines', in the first-stage chapters, to identify causal relationships between independent and dependent variables, thereby constructing a multicausal logical analysis of the economic material under consideration and, at the same time, to attack the indeterminacy of the classical theory. Still, the author of a formidable defense of probable reasoning (of having some reasons to believe, in his own words) against the too strong requirement of certainty of belief, as well as of a logical instead of mathematical concepts of probability, Keynes constantly reminded readers of *The General Theory* of the danger of blind mathematical manipulations, inviting them to avoid reasoning in terms of certainty and mathematical probability, and reiteratedly (sic) insisted on the strong limitations of mathematical analysis." (Carabelli and Cedrini, 2014).

The same confusions and errors made by Backhouse are repeated here.

The claim that

> "Keynes used functional relationships and abstract (i.e. not precisely specified) functions only. Moreover, he used the more precise term 'curve' when exposing the classical theory, but always opted for the more abstract term 'function' when expounding his theory (Backhouse, 2010)." (Carabelli and Cedrini, 2014) is the exact, same error made by Backhouse.

Again, we need to go back to the TP and see what Keynes really said:

> "The hope, which sustained many investigators in the 19th century, of gradually bringing the moral sciences under the sway of mathematical reasoning, steadily recedes—if we mean, as they meant, by mathematics the

introduction of precise numerical methods. The old assumptions, that all quantity is numerical and that all quantitative characteristics are additive, can be no longer sustained. Mathematical reasoning now appears as an aid in its symbolic rather than in its numerical character." (Keynes, 1921).

Carabelli and Cedrini are completely unaware of the type of mathematics being used by Keynes in the TP and GT. Keynes's objection is to the use of mathematics based on the assumptions of additivity and linearity. That is his major objection. Keynes rejected the Marshallian approach to both methodology and mathematics in the GT. Keynes's *A Treatise on Money, volume 2,* (TM, 1930) was mainly based on Marshall's approach to both mathematics and methodology.

It is quite impossible to grasp Keynes's anti –Marshallian view at the GT's macroscopic level if a reader believes that Keynes is using the same tools of analysis he used in the TM:

"Many authors – e.g., Clower (1975), Leijonhufvud (2006), Hayes (2006), and Lawlor (2006)—have defended the view that a correct understanding of Keynes's *General Theory* requires a central place is given to his Marshallian lineage. While agreeing with these authors, we differ from them as far as its implications are concerned. It is true that reading the *General Theory* in this way is enlightening (De Vroey, 2008).

In fact, reading the GT as an application of Marshallian approach to mathematics and methodology, which is based on partial equilibrium, ceteris paribus thinking revolving around an analysis of functions with only one independent variable, is precisely the reason why we still are confronted with the great number of "What did Keynes (really) mean" papers still emanating from the pens of historians of economic thought. It is definitely not only not enlightening, but confusing. It is simply impossible to use the assumptions of partial equilibrium and ceteris paribus at the macro level according to Keynes. Involuntary Unemployment is not generated in the labor market. It is generated at the macroeconomic level if investment spending is deficient.

5. Conclusion

Keynes used the terms "curve " and "function" to describe his own work in the GT He also used these terms to describe the 'classical ' school that he demonstrated mathematically to be a special case of his own GT .. Keynes did not use the word "function" to only describe his more general work. Keynes did not use the word "curve "to only describe the classical use of mathematics. The words "curve" and "function" have been completely misrepresented, misconstrued, misused, and misinterpreted by Backhouse, Carabelli and Cedrini, Wray and others in their assessments of Keynes's approach to the use of mathematics in economics, in general, and especially in their discussions of the role of mathematics in the GT, due to their failure to grasp Keynes's approach to the use of mathematical analysis in economics.

It is quite disconcerting to realize that economists still can't understand Keynes's view on the role of mathematics in economics 82 years after its publication because they still believe that he was a Marshallian. De Vroey's 2008 summary is a perfect example of this erroneous approach:

This paper aimed to assess the implication of trying to anchor Keynes's theory more firmly on Marshallian theory. Many others have emphasized the need for such an anchorage but they have taken for granted that it would reinforce the validity of Keynes's argumentation (De Vroey, 2008).

Of course, what is leads to is complete "nonsense", which was the word chosen by Keynes to describe Joan Robinson's M=L(r), Marshallian analysis of his liquidity preference theory of the rate of interest in a letter to Robinson on November 9th,1936 on page 147 of Volume 14 of the CWJMK. Keynes was not a Marshallian at the time he gave his December 4th,1933 student lecture, where he presented the first IS-LM (LP) model in history. D. Champernowne and W. Brian Reddaway, who attended that lecture, later published versions of IS-LM models based on Keynes's original presentation. IS-LM is not Marshallian.IS-LM is not Walrasian. The only valid interpretation of the GT is Keynes's, which combined IS-LM with the D-Z model. The conclusion, of course, is that Keynes was a Keynesian.

The major finding of the study is that it was Keynes, not Kahn, who developed the mathematical and logical theory of the multiplier in 1921. There were no limitations or problems other than realizing Keynes's unique style involved in presenting mathematical results.

The potential contribution of the paper is that it overturns all existing conclusions currently accepted as fact in the history of economic thought and macroeconomic history as regards the origin and development of the multiplier concept used in the *GT* in chapter 10 in 1936.

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