



The Symmetry Number Structure about Line-1/2

Yajun Liu^{1*} 

¹South China University of Technology, Guangzhou P.R., China 510640. E-mail: yajun@scut.edu.cn

Abstract

Article Info

Volume 5, Issue 1, April 2025

Received : 02 January 2025

Accepted : 05 April 2025

Published : 25 April 2025

doi: [10.51483/IJPAMR.5.1.2025.45-60](https://doi.org/10.51483/IJPAMR.5.1.2025.45-60)

Keywords: *P/2n prime numbers conjectures, Riemann hypothesis*

© 2025 Yajun Liu. This is an open access article under the CC BY license (<https://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

1. The Symmetry of P/2n and Prime Numbers Conjectures

We have P/2n number structure with points [0 1/2N+1 3/4 1] just as shown in Figure1.

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}} & n = 2^N P \\ \frac{3}{4} & n = 2, p = 3 \\ 1 & n = 1, p = 2 \end{cases}$$

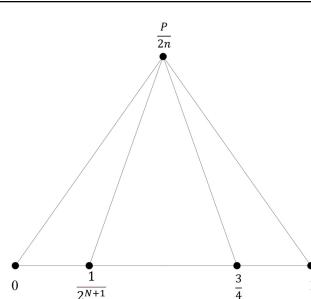


Figure 1: P/2n Number Structure with Points [0, 1/2^{N+1}, 3/4, 1]

* Corresponding author: Yajun Liu, South China University of Technology, Guangzhou P.R., China 510640. E-mail: yajun@scut.edu.cn

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers excepted 0

$P \sim (0, 1, 2, 3, 4, \dots)$ All prime numbers

And

$$\frac{P}{2n} = \begin{cases} \frac{1}{2} - \frac{1}{2n} & P = n-1 (n \geq 3) \\ \frac{1}{2} & P = n \\ \frac{1}{2} + \frac{1}{2n} & P = n+1 \end{cases}$$

And we have

$$p0 \in P \sim (0, n] (n > 2)$$

And based on Bertrand-Chebyshev Theorem: When $n > 2$, there are at least a prime number between n and $2n$.

$$pn \in P \sim [n, 2n] (n > 2)$$

So we have

$$0 < \frac{p0}{2n} \leq \frac{1}{2}$$

$$\frac{1}{2} \leq \frac{pn}{2n} < 1$$

So we have

$$\frac{p0}{2n} = \frac{1}{2} - \frac{1}{2n} (n \geq 3)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

So

$$\left(\frac{1}{2} - \frac{1}{2n} \right) + \left(\frac{1}{2} + \frac{1}{2n} \right) = \frac{p0}{2n} + \frac{pn}{2n}$$

$$2n = p0 + pn (n > 3)$$

This is the proof of Goldbach conjecture

And

$$\frac{pn}{2n} - \frac{p0}{2n} = \left(\frac{1}{2} + \frac{1}{2n} \right) - \left(\frac{1}{2} - \frac{1}{2n} \right)$$

$$pn - p0 = 2$$

This is the proof of Twin Primes Conjecture

And we also have

$$0 < \frac{p0}{2n} = \frac{2k_1 + 1}{2n} \ll 1/2 (n \geq 3)$$

$$0 < 2k_1 + 1 \ll n$$

$$0 < k_1 \ll \frac{n-1}{2}$$

k_1 is a positive integer

$$\text{so } k_1 \sim 1, 2, 3, \dots \left[\frac{n-1}{2} \right] (n \geq 3)$$

And

$$\frac{1}{2} \ll \frac{pn}{2n} = \frac{2k_2 + 1}{2n} < 1$$

$$n \ll 2k_2 + 1 < 2n$$

$$\frac{n-1}{2} \ll k_2 < \frac{2n-1}{2} (n \geq 3)$$

k_2 is a positive integer

$$\text{so } k_2 \sim \left[\frac{n-1}{2} \right], \left[\frac{n-1}{2} \right] + 1, \dots \left[\frac{2n-1}{2} \right] (n \geq 3)$$

we have

$$\frac{pn}{2n} - \frac{p0}{2n} = \frac{2k_2 + 1}{2n} - \frac{2k_1 + 1}{2n}$$

$$pn - p0 = 2(k_2 - k_1)$$

$$(pn - p0)_{\max} = 2(k_2_{\max} - k_1_{\min}) = 2 \left(\left[\frac{2n-1}{2} \right] - 1 \right) = 2(n-2) (n \geq 3)$$

This is the proof of Polignac's conjecture.

So we get a symmetry structure of $P/2n$ as Figure 2

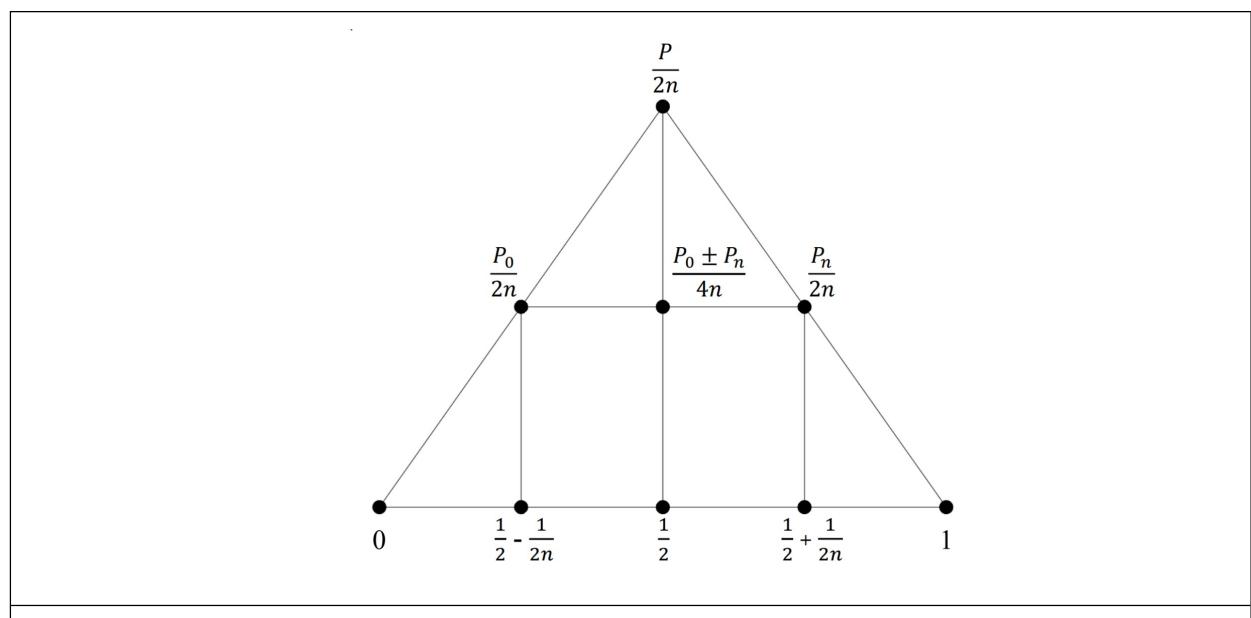


Figure 2: A Symmetry Structure of $P/2n$ about Line-1/2

$$\frac{p0}{2n} = \frac{1}{2} - \frac{1}{2n} \quad (n \geq 3)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

$$\frac{pn + p0}{4n} = \frac{1}{2}$$

2. A Concise Proof of The Fermat' Last Theorem

2.1. The Fermat' Last Theorem

$x^n + y^n = z^n$ ($x, y, z \in n, xyz \neq 0 n > 2$) has no solution.

$n \sim (1, 2, 3, 4, 5, 6, \dots)$ all the natural numbers excepted 0

The equivalent proposition of this conjecture is

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$$

$(x, y, z \in n, xyz \neq 0 n > 2)$ has no solution.

We have

$n \sim (1, 2, 3, 4, 5, 6, \dots)$ all the natural numbers excepted 0

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$$

$$= \left(\frac{1}{2^n} - \frac{1}{2^n}\right) + 1$$

$$= \left(\frac{1}{2} - \frac{1}{2^n}\right) + \left(\frac{1}{2} + \frac{1}{2^n}\right)$$

$$= \frac{1}{2^n} + \left(1 - \frac{1}{2^n}\right)$$

Only when $n = 1$ we have

$$\left(\frac{1}{2^n} - \frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = 0$$

$$1 = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2}\right)$$

And only when $n = 2$

$$\left(\frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = \frac{1}{4}$$

$$\left(1 - \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \frac{3}{4}$$

And we can get the Figures as Figure 3.

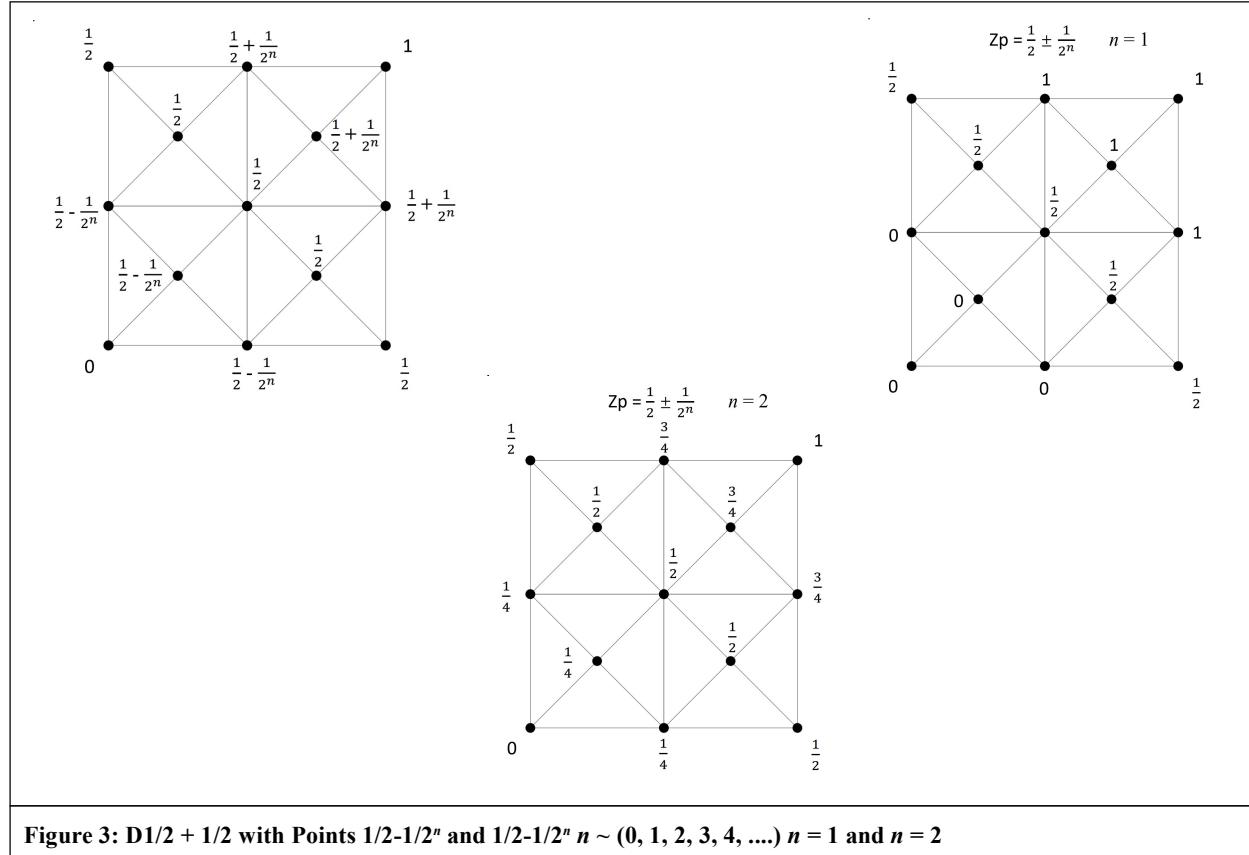


Figure 3: D_{1/2 + 1/2} with Points $1/2 \pm 1/2^n$ and $1/2 \pm 1/2^n$ $n \sim (0, 1, 2, 3, 4, \dots)$ $n = 1$ and $n = 2$

In fact we have

$$1 = \frac{1}{2^1} + \frac{1}{2^1} = \frac{1}{2^2} + \frac{3}{2^2} = \frac{1}{2^3} + \frac{7}{2^3} \text{ or } \frac{3}{2^3} + \frac{5}{2^3}$$

$$\left(\frac{p}{q}\right)^n \quad p, q \text{ is relatively prime and } n \sim (1, 2, 3, 4, \dots)$$

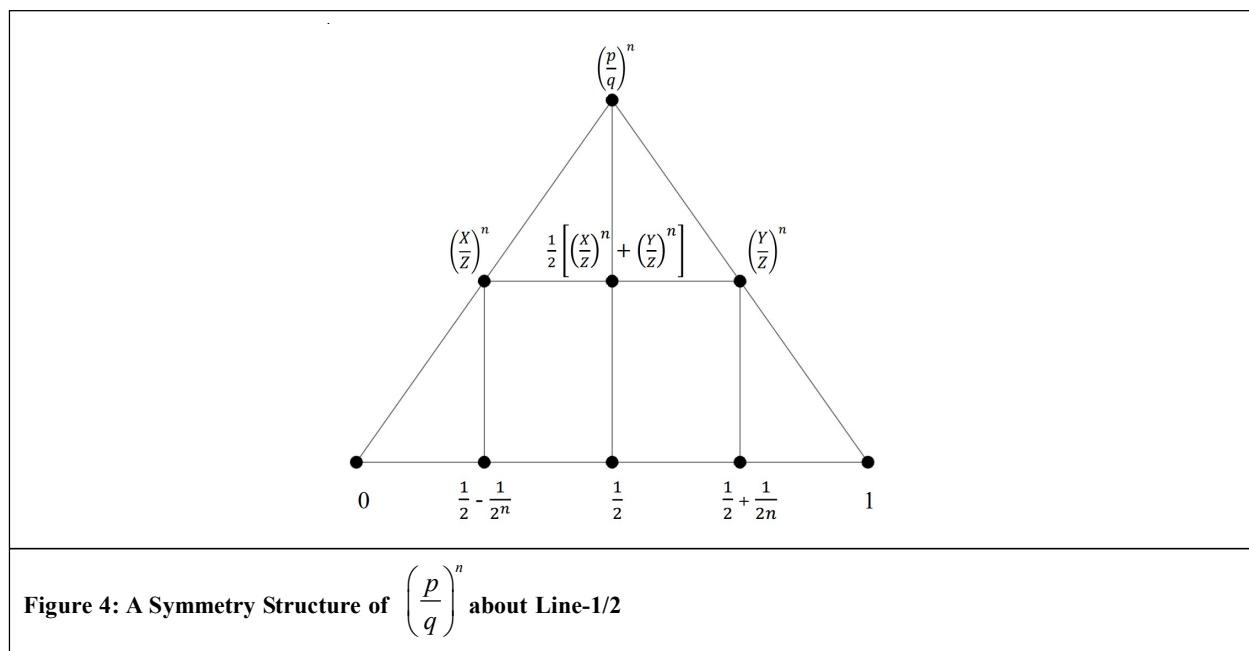


Figure 4: A Symmetry Structure of $\left(\frac{p}{q}\right)^n$ about Line-1/2

$$\frac{1}{2} \left[\left(\frac{x}{z} \right)^n + \left(\frac{y}{z} \right)^n \right] = \frac{1}{2} \Leftrightarrow \left(\frac{x}{z} \right)^n + \left(\frac{y}{z} \right)^n = 1$$

$$\left(\frac{x}{z} \right)^n - \frac{1}{2} - \frac{1}{2^n}$$

$$\left(\frac{y}{z} \right)^n - \frac{1}{2} + \frac{1}{2^n}$$

3. A Concise Proof of Collatz Conjecture

3.1. Collatz Conjecture

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod{2} \\ 3n+1 & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

$$k \in N \rightarrow f^k(n) = 1$$

$n \sim (1, 2, 3, 4, \dots)$ all the natural numbers excepted 0

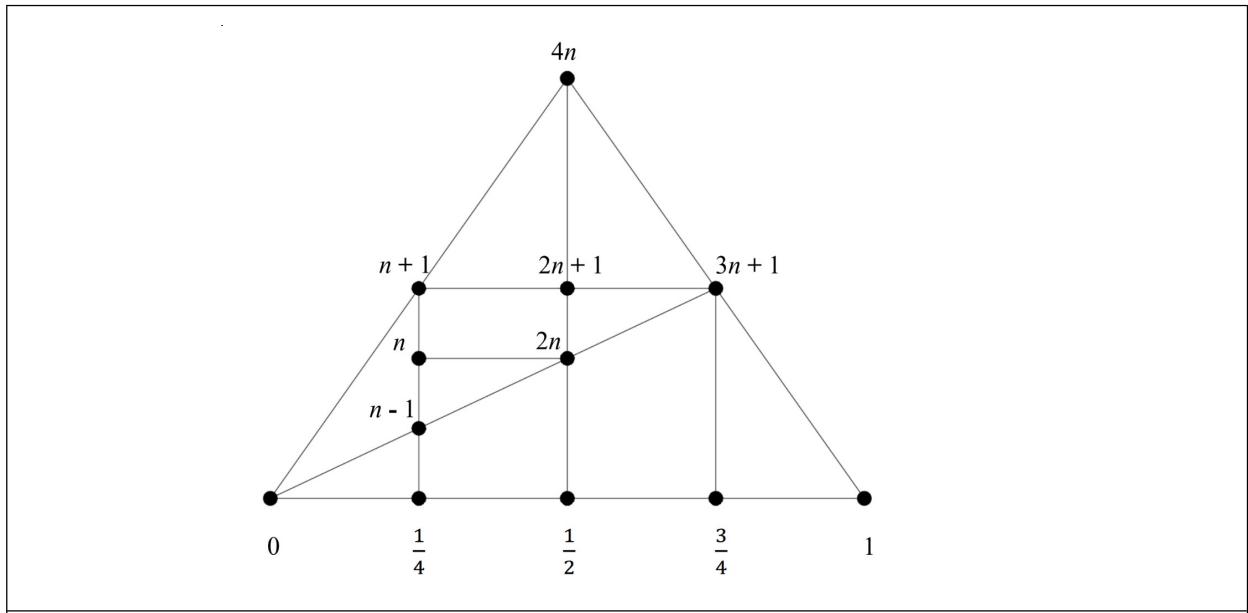


Figure 5: A Symmetrical Structure of $4n$ about Line-1/2

$$2n = 1/2[(n-1) + (3n+1)]$$

$$2n+1 = 1/2[(n+1) + (3n+1)]$$

$$\lim_{n \rightarrow \infty} \left(\frac{n-1}{4n} \right) = 1/4$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+1}{4n} \right) = 1/4$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n+1}{2n} \right) = 1/2$$

$$\lim_{n \rightarrow \infty} \left(\frac{3n+1}{4n} \right) = 3/4$$

$$\frac{\left[\frac{n+1}{2} \right] + \left[\frac{n-1}{2} \right]}{\left[\frac{n}{2} \right]} = \frac{(n-1)+(3n+1)}{2n} = \frac{(n+1)+(3n+1)}{2n+1} = \frac{4n}{2n} = \frac{4}{2}$$

$$= \frac{2}{1} = \frac{1}{\frac{1}{2}} = \sum \frac{1}{2^N}$$

This is a concise proof of $3n + 1$ Conjecture.

In fact, we can get a symmetrical structure of $4n$ about line-1/2 just as Figures 6 and 7.

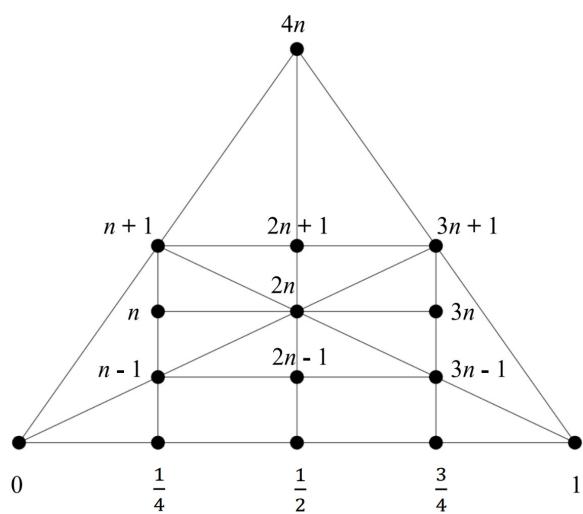


Figure 6: A Symmetrical Structure of $4n$ about Line-1/2

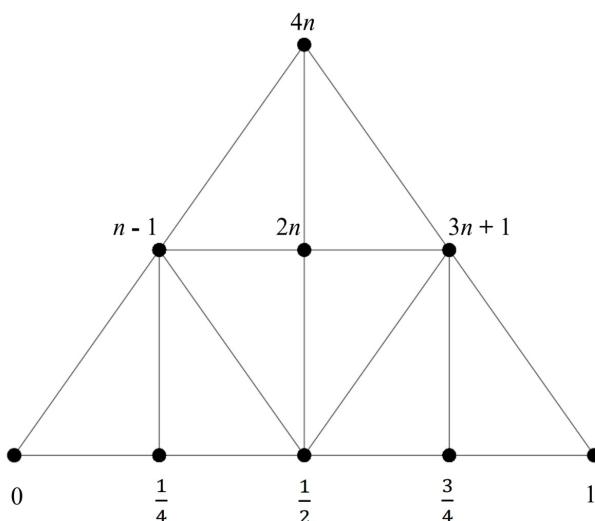


Figure 7: A Symmetrical Structure of $4n$ about Line-1/2

4. The Proof of Riemann Hypothesis

4.1. Riemann Zeta-Function

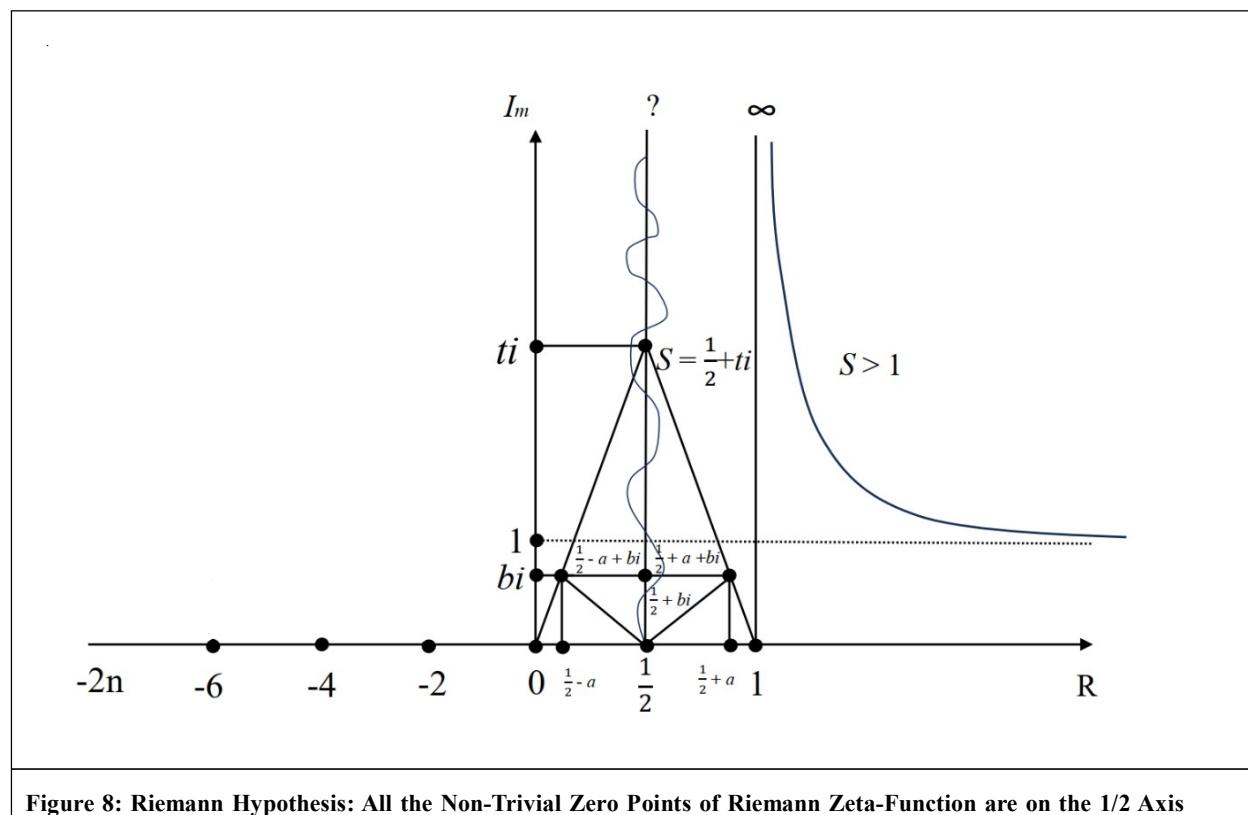
$$\xi(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod \frac{1}{1 - p^{-s}} (s = a + bi)$$

$$S > 1 \quad \xi(s) \rightarrow const$$

The trivial zero-points of Riemann Zeta-Function is $-2n$ ($n \sim 1, 2, 3, \dots$)

Riemann Hypothesis: All the non-trivial zero-point of Zeta-Function $\operatorname{Re}(s) = \frac{1}{2}$.

We can get a symmetrical structure including all numbers about the line-1/2 as Figure 8.



$$s = \frac{1}{2} + ti \quad t \in R$$

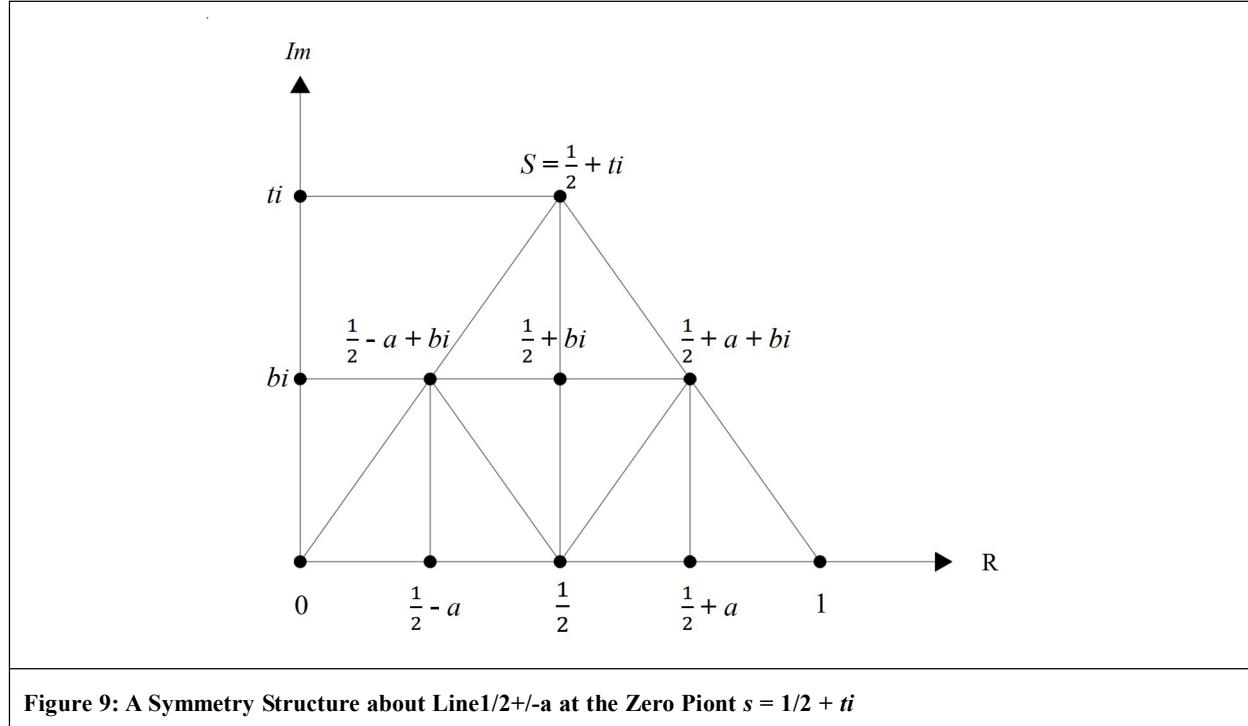
$$zp1 = \frac{1}{2} - a + bi \quad zp0 = \frac{1}{2} + bi \quad zp2 = \frac{1}{2} + a + bi$$

$$zp1 + zp2 = \left(\frac{1}{2} - a + bi \right) + \left(\frac{1}{2} + a + bi \right) = 1 + 2bi$$

$$zp2 - zp1 = \left(\frac{1}{2} + a + bi \right) - \left(\frac{1}{2} - a + bi \right) = 2a$$

$$a, b \in R \quad 0 \leq a \leq \frac{1}{2}$$

As the Figure 9. If we have zero points of $\tilde{\alpha}(s)$ on line- $1/2 \pm a$ as

Figure 9: A Symmetry Structure about Line $1/2 \pm a$ at the Zero Point $s = 1/2 + ti$

$$zp1 = \frac{1}{2} - a + bi \quad zp2 = \frac{1}{2} + a + bi$$

And $s = \frac{1}{2} + ti$ $t \in R$ is the first zero point on line- $1/2$

So

$$\xi\left(\frac{1}{2} - a + bi\right) = 0$$

$$\xi\left(\frac{1}{2} + a + bi\right) = 0$$

$$\xi\left(\frac{1}{2} - a + bi\right) * \xi\left(\frac{1}{2} + a + bi\right) = \sum_{n=1} \frac{1}{\left[\left(\frac{1}{2} - a + bi\right) + \left(\frac{1}{2} + a + bi\right)\right]} = \xi^2\left(\frac{1}{2} + bi\right) = 0$$

$$\xi\left(\frac{1}{2} + bi\right) = 0$$

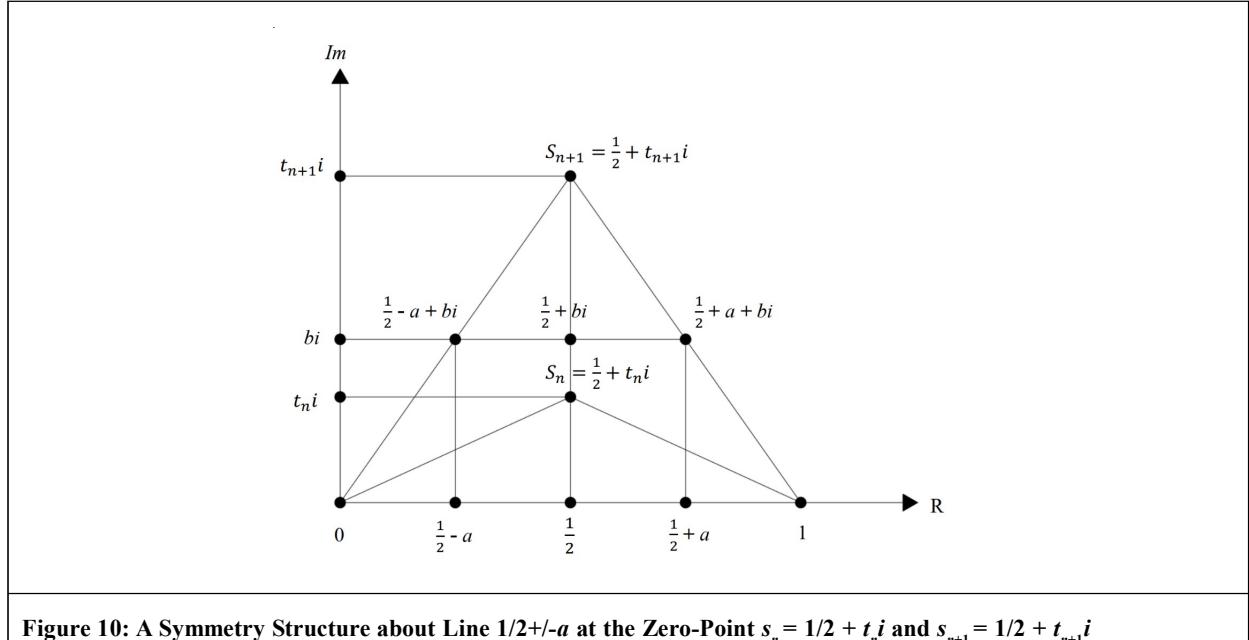
So we can get a zero point of $\xi(s)$ as

$$zp0 = \frac{1}{2} + bi \quad 0 < b < t \quad b, t \in R$$

It is contrary to that $s = \frac{1}{2} + ti$ $t \in R$ is the first zero point on line- $1/2$

As the Figure 10. If we have zero points of $\xi(s)$ on line $1/2 \pm a$ as

$$zp1 = \frac{1}{2} - a + bi \quad zp2 = \frac{1}{2} + a + bi$$

**Figure 10: A Symmetry Structure about Line $1/2+/-a$ at the Zero-Point $s_n = 1/2 + t_n i$ and $s_{n+1} = 1/2 + t_{n+1} i$**

And $s_n = \frac{1}{2} + t_n i$ $t \in R$ is the No. n zero point on line- $1/2$

$s_{n+1} = \frac{1}{2} + t_{n+1} i$ $t \in R$ is the No. $n+1$ zero point on line- $1/2$

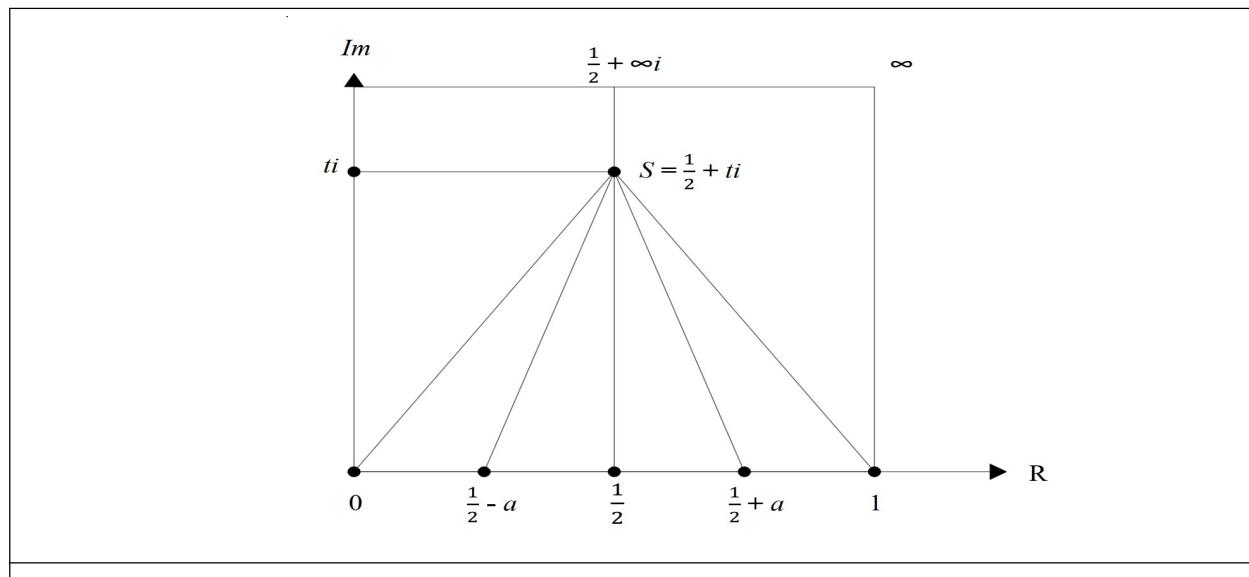
We can get a zero point of $\xi(s)$ between s_n and s_{n+1} on line- $1/2$ as

$$zp0 = \frac{1}{2} + bi \quad t_n < b < t_{n+1} \quad b, t \in R$$

It is contrary to that s_n and s_{n+1} are the adjacent zero points on line- $1/2$

So on complex plane, we can have the symmetry structure about the line- $1/2$ with $zp = 1/2 \pm a$ $\left(0 \leq a \leq \frac{1}{2}, a \in R \right)$

show as on Figure 11.

**Figure 11: Symmetry Structure about the Line- $1/2$ with $zp = 1/2 \pm a$**

$$S = \frac{1}{2} + ti \quad (t \in R)$$

$$zp = \frac{1}{2} \pm a \left(0 \leq a \leq \frac{1}{2}, a \in R \right)$$

$$zp1 = \frac{1}{2} - a \quad zp0 = \frac{1}{2} \quad zp2 = \frac{1}{2} + a$$

$$zp1 + zp2 = \left(\frac{1}{2} - a \right) + \left(\frac{1}{2} + a \right) = 1$$

$$zp2 - zp1 = \left(\frac{1}{2} + a \right) - \left(\frac{1}{2} - a \right) = 2a$$

This is mean that there are no zero points on line- $1/2 \pm a$ ($0 \leq a \leq \frac{1}{2}, a \in R$)

Hardy and Littlewood (1914) give a proof that there are infinite zero points on line- $1/2$.

So we give a proof that all the non-trivial Zero points of Riemann zeta-function are on the Line- $1/2$.

This is the proof of Riemann Hypothesis.

5. The Symmetry Number Structure about Line- $1/2$ Including All Numbers

In fact, we have a symmetrical number structure about line- $1/2$ as Figure 12.

And we can get a symmetrical number structure about line- $1/2$ as Figure 13. We should call it Reimann dynamic space.

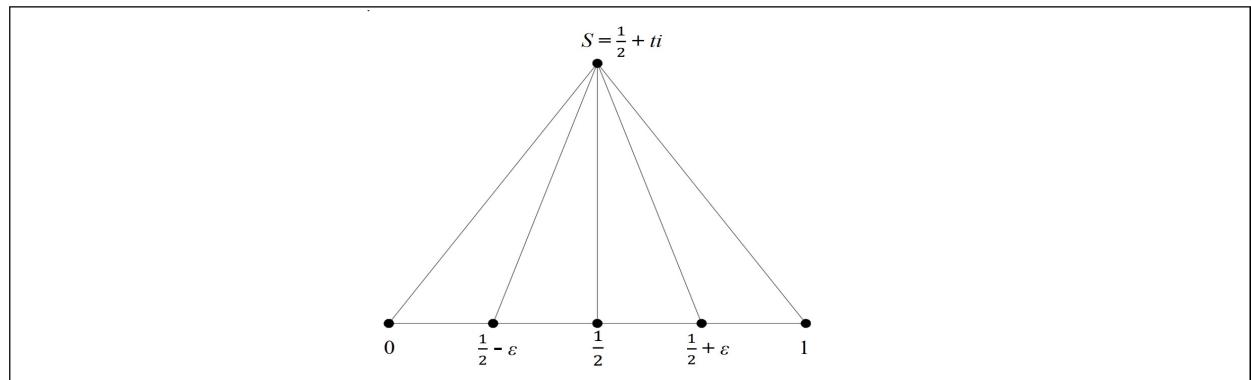


Figure 12: Symmetry Structure about the Line- $1/2$ with $zp = 1/2 \pm \varepsilon$

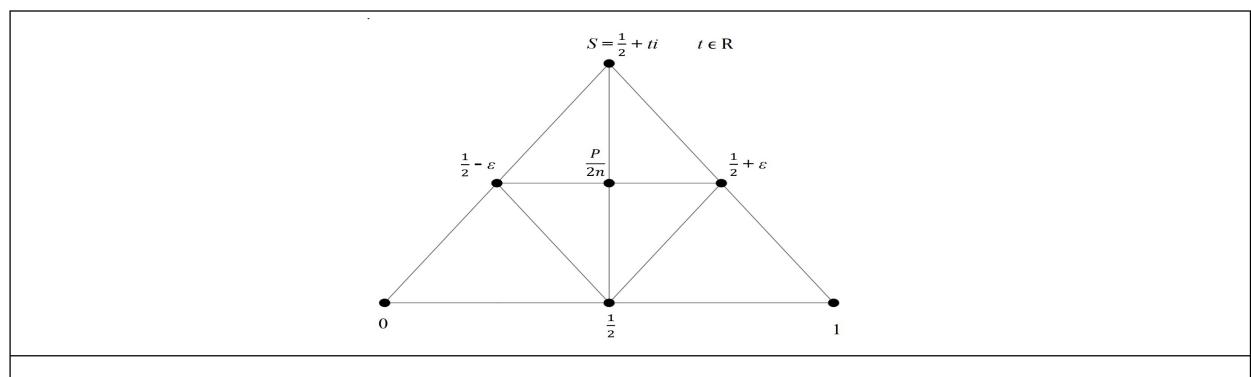


Figure 13: Reimann Dynamic Space

$$1 + i^2 = 0$$

$$1 + 1/2(i+1)(i-1) = 0$$

$$S = \frac{1}{2} + ti \quad (t \in R)$$

$$zp = \frac{1}{2} \pm \varepsilon \left(\varepsilon = a + bi \quad a, b \in R \quad 0 \leq a \leq \frac{1}{2} \right)$$

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}} & n = 2^N P \\ \frac{3}{4} & n = 2 \quad P = 3 \\ 1 & n = 1 \quad P = 2 \end{cases}$$

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, \dots)$ All prime numbers

We can have point $p1, p2, p3, p4$ and

$$p1 \in \left(\frac{1}{2} - a, \frac{p}{2n} \right)$$

$$p2 \in \left(\frac{p}{2n}, \frac{1}{2} + a \right)$$

$$p1 + p2 = \frac{p}{n}$$

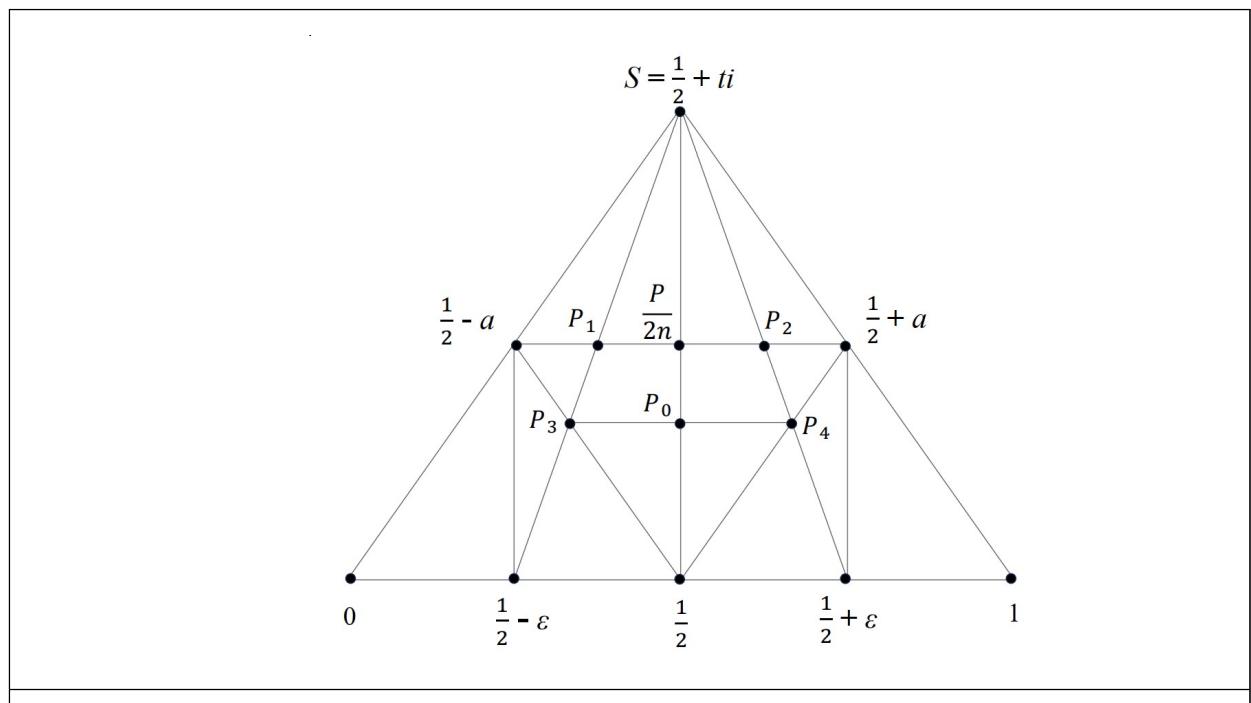


Figure 14: Reimann Dynamic Space with $p1, p2, p3, p4$

$$p3 \in \left(\frac{1}{2} - a, \frac{1}{2} \right)$$

$$p4 \in \left(\frac{1}{2}, \frac{1}{2} + a \right)$$

$$p0 = 1/2(p3 + p4)$$

And we can get Figure 15.

$$1. \quad zp = \frac{1}{2} + bi \quad 0 < b < t \quad b, t \in R \quad (\text{the proof of RH})$$

$$2. \quad \left(\frac{x}{z} \right)^n + \left(\frac{y}{z} \right)^n = 1$$

$$\left(\frac{x}{z} \right)^n \leftrightarrow \frac{1}{2} - \frac{1}{2^n}$$

$$\left(\frac{y}{z} \right)^n \leftrightarrow \frac{1}{2} + \frac{1}{2^n} \quad (\text{the proof of F.L.T})$$

$$3. \quad \frac{p0}{2n} \leftrightarrow \frac{1}{2} - \frac{1}{2n}$$

$$\frac{pn}{2n} \leftrightarrow \frac{1}{2} + \frac{1}{2n} \quad (\text{the proof of GC/BC/TPC})$$

$$\frac{pn \pm p0}{4n} \leftrightarrow \frac{1}{2}$$

$$4. \quad 4n = 2n + 2n = [(n-1) + (3n+1)]$$

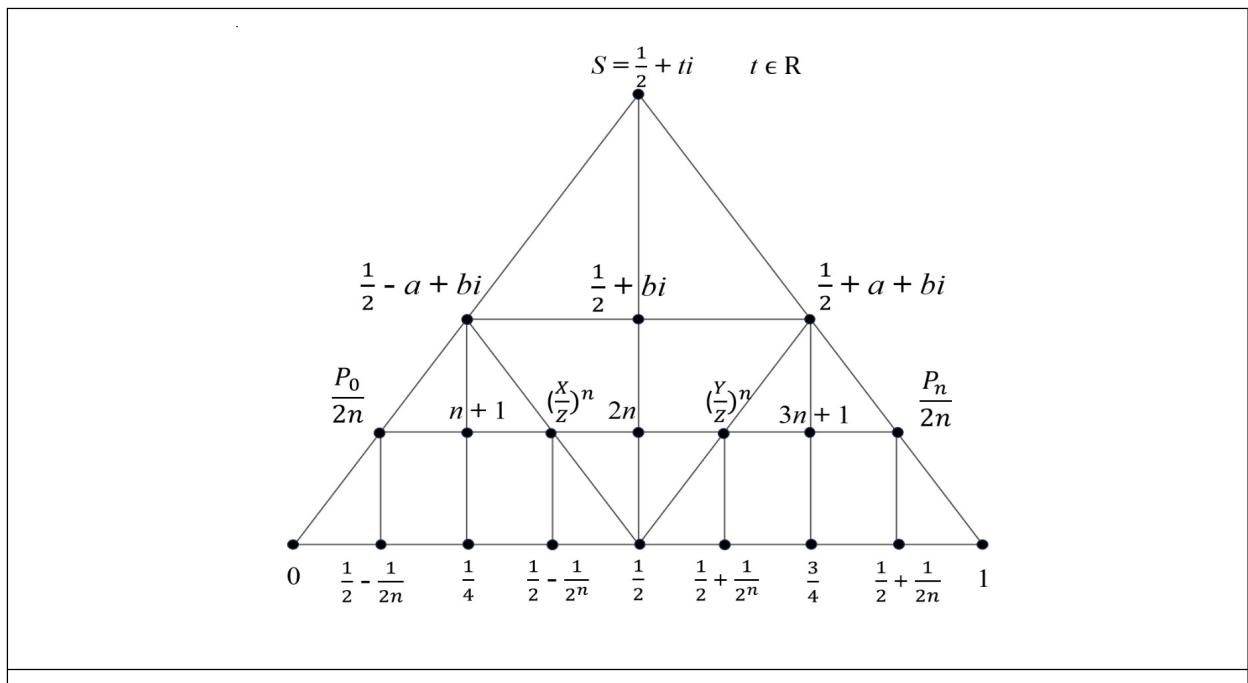


Figure 15: Reimann Dynamic Space and Number Conjectures

$$\lim_{n \rightarrow \infty} \left(\frac{n-1}{4n} \right) = 1/4$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+1}{4n} \right) = 1/4 \text{ (the proof of } 3n+1 \text{ conjecture)}$$

And we have

$$1/2 = 1/2, 0 = 1/2 - 1/2, 1 = 1/2 + 1/2$$

$$1 + (+i)^2 = 0$$

$$1 + 1/2(i+1)(i-1) = 0$$

$$\infty = 1+1+1+1+\dots$$

We called it $L^{1/2 \pm \varepsilon}[0 \ 1/2 \ 1]$ and analytic continuation to $\begin{bmatrix} +\infty & -\infty \\ -\infty & +\infty \end{bmatrix}$ we can get Figure 16.

So we have:

$$1 + \begin{bmatrix} +\infty & i & -\infty \\ 0 & 1/2 & 1 \\ -\infty & -i & +\infty \end{bmatrix} \begin{bmatrix} 1 & 1/2 & 0 \\ \frac{1}{2} - \varepsilon & 1/2 & \frac{1}{2} + \varepsilon \\ 1 & 1/2 & 0 \end{bmatrix}^{-1} = 0$$

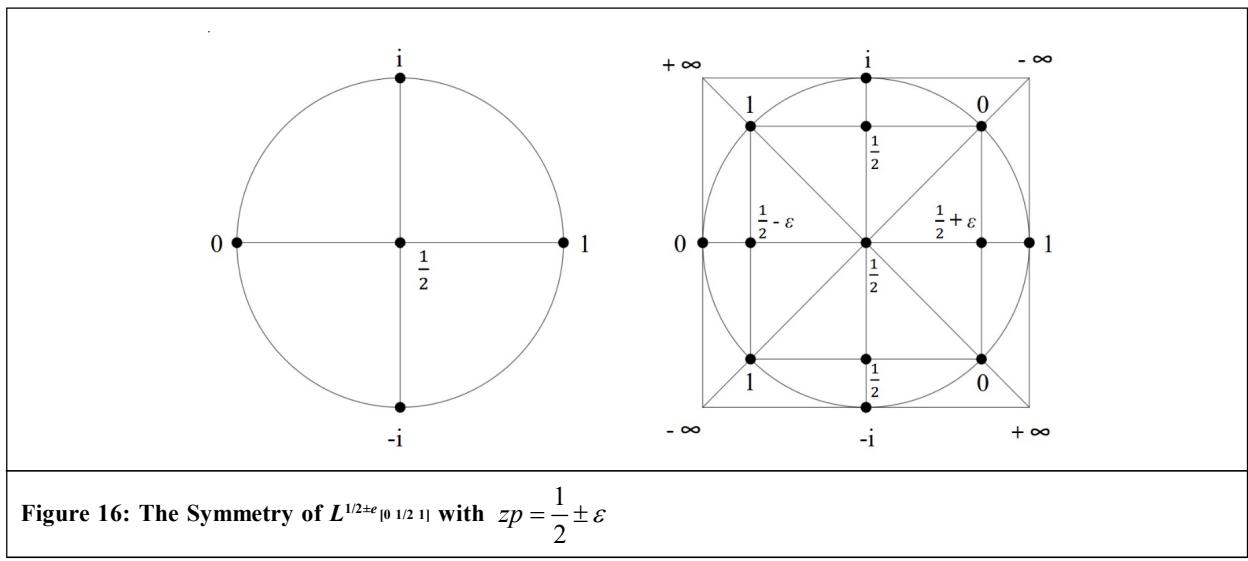
$$zp = \frac{1}{2} \pm \varepsilon$$

$$\varepsilon = a + bi \quad (a, b \in R, 0 \leq a \leq \frac{1}{2})$$

$$zp1 = \frac{1}{2} - \varepsilon, zp2 = \frac{1}{2} + \varepsilon$$

We have

$$zp1 + zp2 = \left(\frac{1}{2} - \varepsilon \right) + \left(\frac{1}{2} + \varepsilon \right) = 1$$



$$zp2 - zp1 = \left(\frac{1}{2} + \varepsilon\right) - \left(\frac{1}{2} - \varepsilon\right) = 2\varepsilon = 2(a + bi)$$

And we have

$$n^2 = \frac{1}{2} \cdot n \cdot 2n = \sum_{k=1}^N \frac{1}{2} \sum_{j=1}^N \frac{1}{2^N} \left[\left(\frac{1}{2} - \varepsilon\right) + \left(\frac{1}{2} + \varepsilon\right) \right]$$

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

We can get a matrix ($n \times n$)

$$\begin{bmatrix} 1/2 & \cdots & \frac{1}{2^n}(1/2 + \varepsilon) \\ \cdots & 1/2 & \cdots \\ \frac{1}{2^n}(1/2 - \varepsilon) & \cdots & 1/2 \end{bmatrix} (n \times n)$$

The $tr(A) = 1/2 * n$

We have

$$0 = \frac{1}{2} - \frac{1}{2}, 1 = \frac{1}{2} + \frac{1}{2}, 2 = 1 + 1$$

$$1 + i^2 = 0, 1/2 + i^{4N+1} = 1/2 + i$$

$$\infty = 1 + 1 + 1 + \cdots$$

$$p_0 \in P < 2n, p_n \in P > 2n$$

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$n \sim (1, 2, 3, 4, \dots)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, \dots)$ All odd prime numbers

$$S = \frac{1}{2} + t (t \in R)$$

$$zp = \frac{1}{2} \pm \varepsilon \left(\varepsilon = a + bi, a, b \in R, 0 \leq a \leq \frac{1}{2} \right)$$

And we find that

1. $1 + e^{\pi i} = 0$ (Euler's Formula)

$$1 + i^2 = 0, 1 + \frac{1}{2}(i+1)(i-1) = 0, (1+i)(1-i) = \sum \frac{1}{2^N}$$

$$1 + e^{\pi i} = 0, 1 + \frac{1}{2}(e^{ip\pi} - e^{i2N\pi}) = 0$$

$N \sim (0, 1, 2, 3, 4, \dots)$ All natural numbers

$p \sim (3, 5, 7, \dots)$ All odd prime numbers

2. $2(n+1) = pn + p_0$

$$pn - 2n + p_0 = 2$$

And

$$2n - pn + p0 = 2$$

It is like the Euler's Polyhedron Formula

We can get Figure 17. This is a symmetry number structure about line-1/2 including all numbers and the equivalence structure of $S^\infty + i$ is shown as Figure 18.

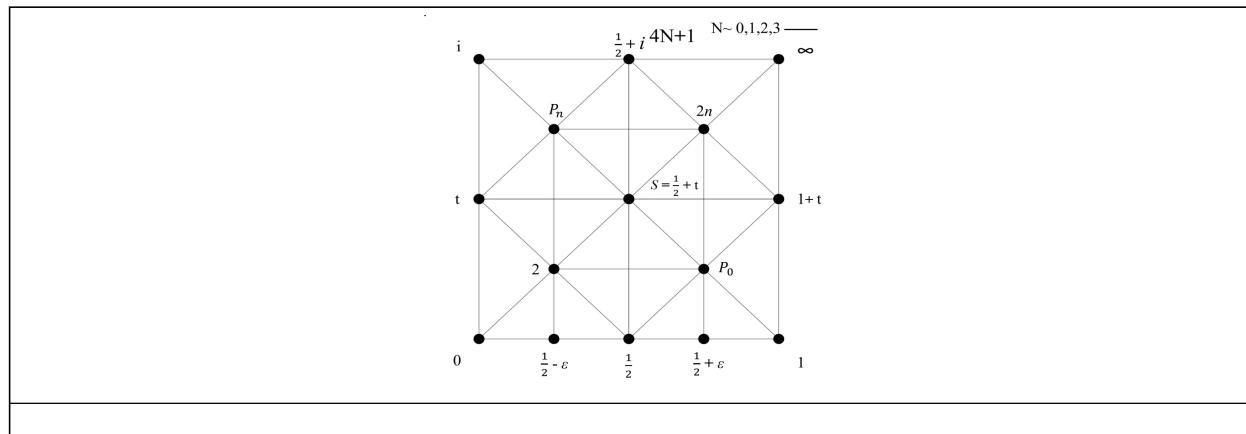


Figure 17: The Symmetry of $S^\infty + i$

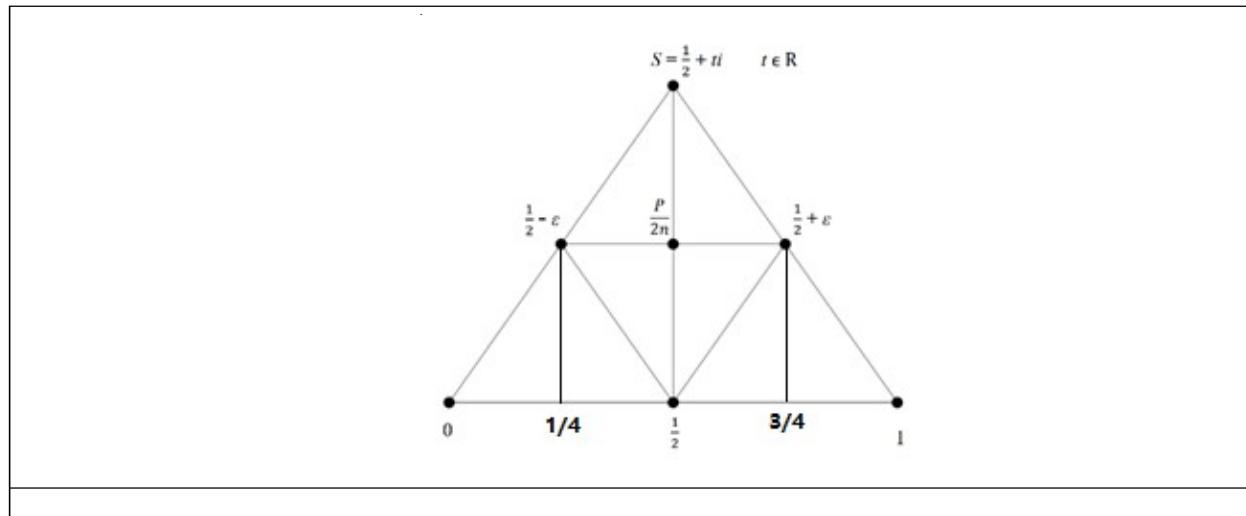


Figure 18: The Equivalence Structure of $S^\infty + i$

References

- Hardy, G.H. and Littlewood, J.E. (1914). Some Problems of Diophantine Approximation: Part I. The Fractional Part of n_k . *Acta Math.* 37, 155-191. doi: 10.1007/BF02401833
- Weisstein, Eric W. (2002). Collatz Problem. *From Math World—A Wolfram Web Resource*. <https://mathworld.wolfram.com/CollatzProblem.html>
- Weisstein, Eric W. (2004). Fermat Last Theorem. *From Math World—A Wolfram Web Resource*. <https://mathworld.wolfram.com/FermatLastTheorem.html>
- Weisstein, Eric W. (2002). Goldbach Conjecture. *From Math World—A Wolfram Web Resource*. <https://mathworld.wolfram.com/GoldbachConjecture.html>
- Weisstein, Eric W. (2003). Riemann Hypothesis. *From Math World—A Wolfram Web Resource*. <https://mathworld.wolfram.com/RiemannHypothesis.html>