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The Symmetry Number Structure about Line-1/2

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Abstract

In this paper, we discuss the symmetry number structure about line-1/2. We find that using the symmetry characters of those structures we can give proofs of the number Conjectures: Goldbach Conjecture Twins Prime Conjecture and Polignac's conjecture and the Riemann Hypothesis. In this paper, we also gave concise proofs of the Fermat' Last Theorem and the $3n + 1$ conjecture.

Keywords: *P/2n prime numbers conjectures, Riemann hypothesis*

1. The Symmetry of P/2*n* and Prime Numbers Conjectures

We have P/2n number structure with points [0 $1/2^{N+1}$ 3/4 1] just as shown in Figure1.

$$\frac{P}{2n}=\begin{cases}\dfrac{1}{2^{N+1}} & n=2^N P\\[4pt] \dfrac{3}{4} & n=2\ \ p=3\\[4pt] 1 & n=1\ \ p=2\end{cases}$$

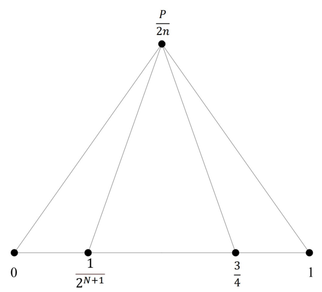

Figure 1: P/2*n* Number Structure with Points [0, $1/2^{N+1}$, 3/4, 1]

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$N \sim (0, 1, 2, 3, 4, ...)$ All natural numbers

$n \sim (0, 1, 2, 3, 4, ...)$ All natural numbers excepted 0

$P \sim (0, 1, 2, 3, 4, ...)$ All prime numbers

And

$$\frac{P}{2n} = \begin{cases} \frac{1}{2} - \frac{1}{2n} & P = n-1\left(n \geq 3\right) \\ \frac{1}{2} & P = n \\ \frac{1}{2} + \frac{1}{2n} & P = n+1 \end{cases}$$

And we have

$p0 \in P \sim (0, n]\ (n > 2)$

And based on Bertrand-Chebyshev Theorem: When $n > 2$, there are at least a prime number between n and $2n$.

$pn \in P \sim [n, 2n)\ (n > 2)$

So we have

$$0 < \frac{p0}{2n} \leq \frac{1}{2}$$

$$\frac{1}{2} \leq \frac{pn}{2n} < 1$$

So we have

$$\frac{p0}{2n} = \frac{1}{2} - \frac{1}{2n}\left(n \geq 3\right)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

So

$$\left(\frac{1}{2} - \frac{1}{2n}\right) + \left(\frac{1}{2} + \frac{1}{2n}\right) = \frac{p0}{2n} + \frac{pn}{2n}$$

$2n = p0 + pn\ (n > 3)$

This is the proof of Goldbach conjecture

And

$$\frac{pn}{2n} - \frac{p0}{2n} = \left(\frac{1}{2} + \frac{1}{2n}\right) - \left(\frac{1}{2} - \frac{1}{2n}\right)$$

$pn - p0 = 2$

This is the proof of Twin Primes Conjecture

And we also have

$$0 < \frac{p0}{2n} = \frac{2k_1 + 1}{2n} \ll 1/2\left(n \geq 3\right)$$

$0 < 2k1 + 1 << n$

$0 < k_1 \ll \frac{n-1}{2}$

k_1 is a positive integer

so $k_1 \sim 1, 2, 3, \ldots \left[\frac{n-1}{2}\right](n \geq 3)$

And

$\frac{1}{2} \ll \frac{pn}{2n} = \frac{2k_2+1}{2n} < 1$

$n \ll 2k_2 + 1 < 2n$

$\frac{n-1}{2} \ll k_2 < \frac{2n-1}{2}(n \geq 3)$

k_2 is a positive integer

so $k_2 \sim \left[\frac{n-1}{2}\right], \left[\frac{n-1}{2}\right]+1, \ldots \left[\frac{2n-1}{2}\right](n \geq 3)$

we have

$\frac{pn}{2n} - \frac{p0}{2n} = \frac{2k_2+1}{2n} - \frac{2k_1+1}{2n}$

$pn - p0 = 2(k2 - k1)$

$(pn - p0)_{\max} = 2(k2_{\max} - k1_{\min}) = 2\left(\left[\frac{2n-1}{2}\right] - 1\right) = 2(n-2)(n \geq 3)$

This is the proof of Polignac's conjecture.

So we get a symmetry structure of P/2*n* as Figure 2

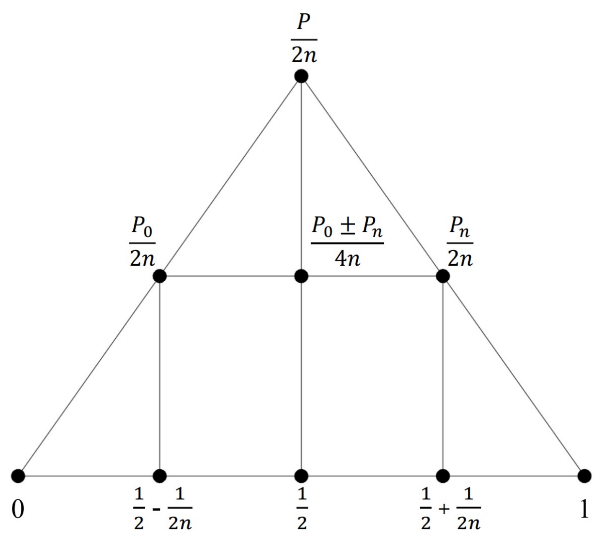

Figure 2: A Symmetry Structure of P/2*n* about Line-1/2

$$\frac{p0}{2n} = \frac{1}{2} - \frac{1}{2n}(n \geq 3)$$

$$\frac{pn}{2n} = \frac{1}{2} + \frac{1}{2n}$$

$$\frac{pn + p0}{4n} = \frac{1}{2}$$

2. A Concise Proof of The Fermat' Last Theorem

2.1. The Fermat' Last Theorem

$x^n + y^n = z^n\left(x,\ y,\ z \in n,\ xyz \neq 0\ n > 2\right)$ has no solution.

$n \sim (1, 2, 3, 4, 5, 6, ...)$ all the natural numbers excepted 0

The equivalent proposition of this conjecture is

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$$

$\left(x,\ y,\ z \in n,\ xyz \neq 0\ n > 2\right)$ has no solution.

We have

$n \sim (1, 2, 3, 4, 5, 6, ...)$ all the natural numbers excepted 0

$$\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$$

$$= \left(\frac{1}{2^n} - \frac{1}{2^n}\right) + 1$$

$$= \left(\frac{1}{2} - \frac{1}{2^n}\right) + \left(\frac{1}{2} + \frac{1}{2^n}\right)$$

$$= \frac{1}{2^n} + \left(1 - \frac{1}{2^n}\right)$$

Only when $n = 1$ we have

$$\left(\frac{1}{2^n} - \frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = 0$$

$$1 = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2}\right)$$

And only when $n = 2$

$$\left(\frac{1}{2^n}\right) = \left(\frac{1}{2} - \frac{1}{2^n}\right) = \frac{1}{4}$$

$$\left(1 - \frac{1}{2^n}\right) = \left(\frac{1}{2} + \frac{1}{2^n}\right) = \frac{3}{4}$$

And we can get the Figures as Figure 3.

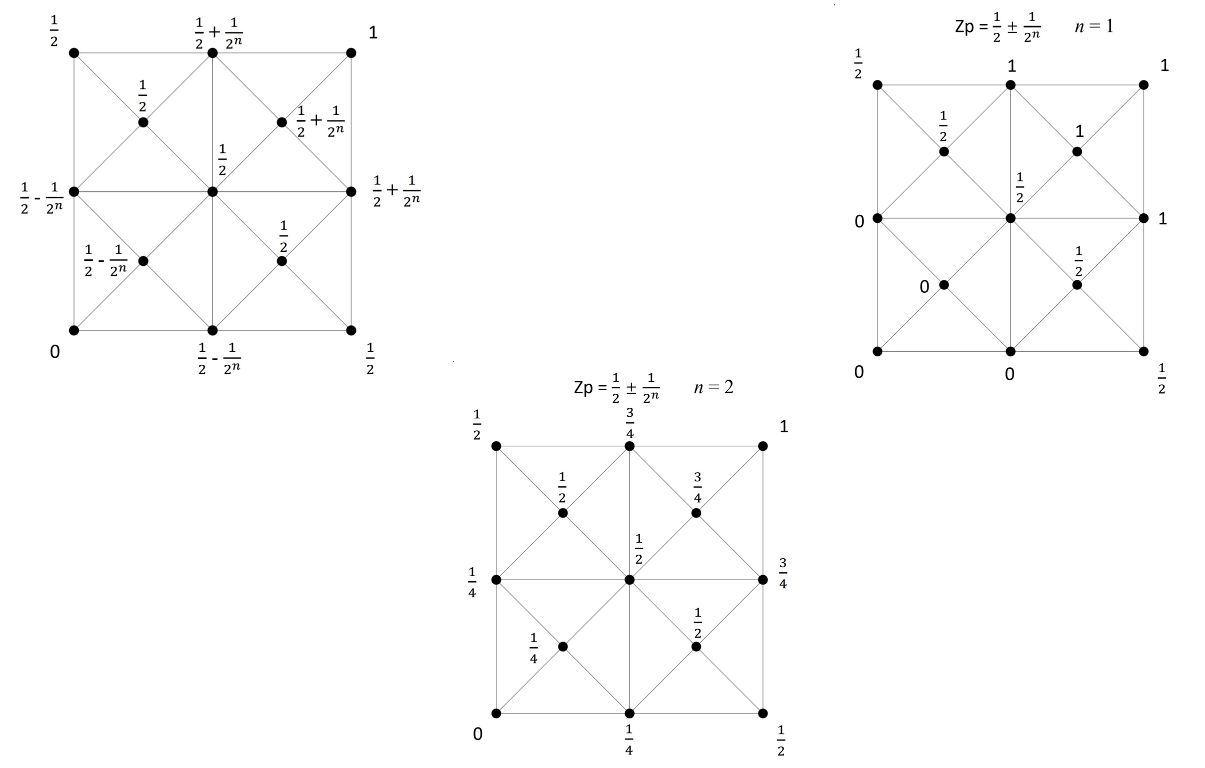

Figure 3: D1/2 + 1/2 with Points 1/2-1/2^n and 1/2-1/2^n n ~ (0, 1, 2, 3, 4, ...) n = 1 and n = 2

In fact we have

$$1 = \frac{1}{2^1} + \frac{1}{2^1} = \frac{1}{2^2} + \frac{3}{2^2} = \frac{1}{2^3} + \frac{7}{2^3} \ or \ \frac{3}{2^3} + \frac{5}{2^3}$$

$\left(\frac{p}{q}\right)^n$ p, q is relatively prime and $n \sim (1, 2, 3, 4, ...)$

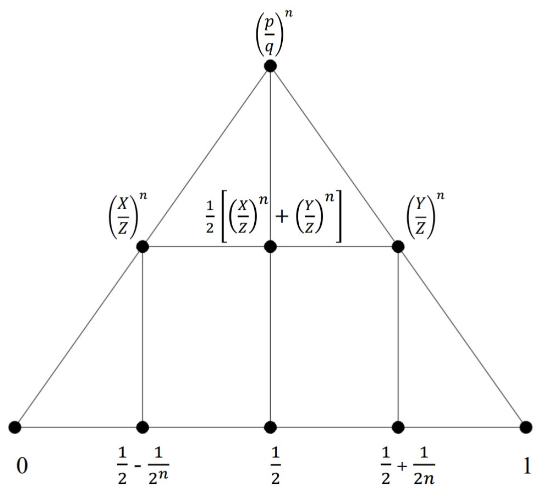

Figure 4: A Symmetry Structure of $\left(\frac{p}{q}\right)^n$ about Line-1/2

$$1/2\left[\left(\frac{x}{z}\right)^n+\left(\frac{y}{z}\right)^n\right]=1/2\leftrightarrow\left(\frac{x}{z}\right)^n+\left(\frac{y}{z}\right)^n=1$$

$$\left(\frac{x}{z}\right)^n-\frac{1}{2}-\frac{1}{2^n}$$

$$\left(\frac{y}{z}\right)^n-\frac{1}{2}+\frac{1}{2^n}$$

3. A Concise Proof of Collatz Conjecture

3.1. Collatz Conjecture

$$f(n)=\begin{cases}\dfrac{n}{2} & if\ \ n\equiv 0(mod2)\\ 3n+1 & if\ \ n\equiv 1(mod2)\end{cases}$$

$k\in N\rightarrow f^k(n)=1$

$n\sim(1,2,3,4,...)$ all the natural numbers excepted 0

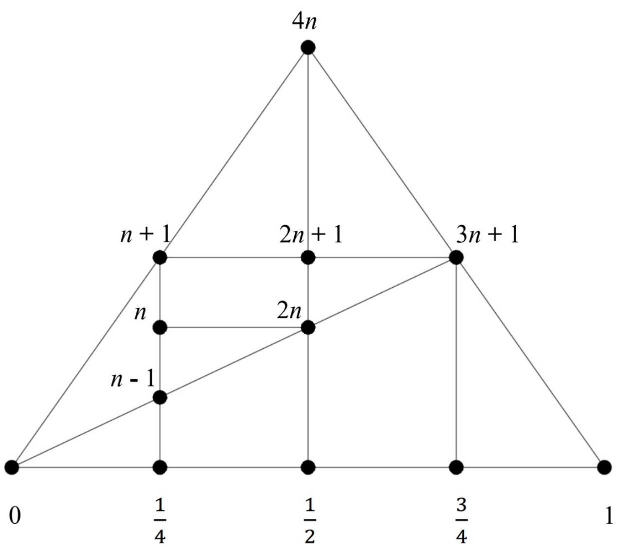

Figure 5: A Symmetrical Structure of 4*n* about Line-1/2

$2n=1/2[(n-1)+(3n+1)]$

$2n+1=1/2[(n+1)+(3n+1)]$

$$\lim_{n\to\infty}\left(\frac{n-1}{4n}\right)=1/4$$

$$\lim_{n\to\infty}\left(\frac{n+1}{4n}\right)=1/4$$

$$\lim_{n\to\infty}\left(\frac{2n+1}{2n}\right)=1/2$$

$$\lim_{n\to\infty}\left(\frac{3n+1}{4n}\right)=3/4$$

$$\frac{\left[\frac{n+1}{2}\right]+\left[\frac{n-1}{2}\right]}{\left[\frac{n}{2}\right]}=\frac{(n-1)+(3n+1)}{2n}=\frac{(n+1)+(3n+1)}{2n+1}=\frac{4n}{2n}=\frac{4}{2}$$

$$=\frac{2}{1}=\frac{1}{\frac{1}{2}}=\sum\frac{1}{2^N}$$

This is a concise proof of $3n + 1$ Conjecture.

In fact, we can get a symmetrical structure of $4n$ about line-1/2 just as Figures 6 and 7.

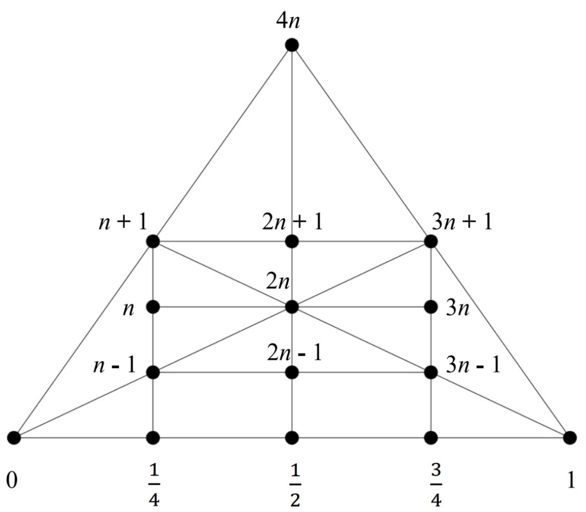

Figure 6: A Symmetrical Structure of 4n about Line-1/2

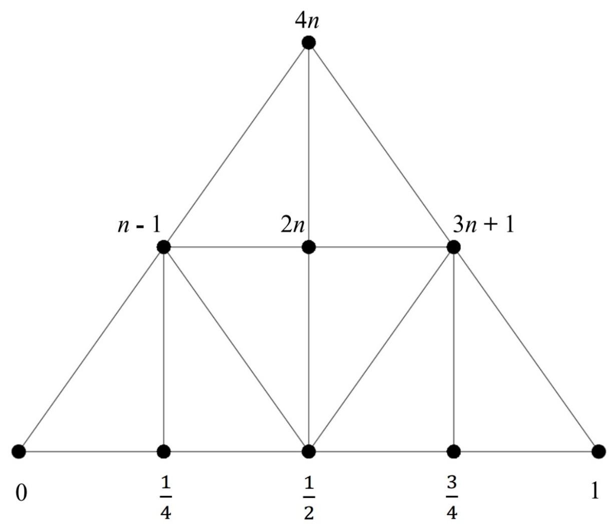

Figure 7: A Symmetrical Structure of 4*n* about Line-1/2

4. The Proof of Riemann Hypothesis

4.1. Riemann Zeta-Function

$$\xi(s)=\sum_{n=1}\frac{1}{n^s}=\prod\frac{1}{1-p^s}\left(s=a+bi\right)$$

$S>1\ \xi(s)\to const$

The trivial zero-points of Riemann Zeta-Function is $-2n$ ($n\sim 1, 2, 3, \ldots$)

Riemann Hypothesis: All the non-trivial zero-point of Zeta-Function $\mathrm{Re}(s)=\frac{1}{2}$.

We can get a symmetrical structure including all numbers about the line-1/2 as Figure 8.

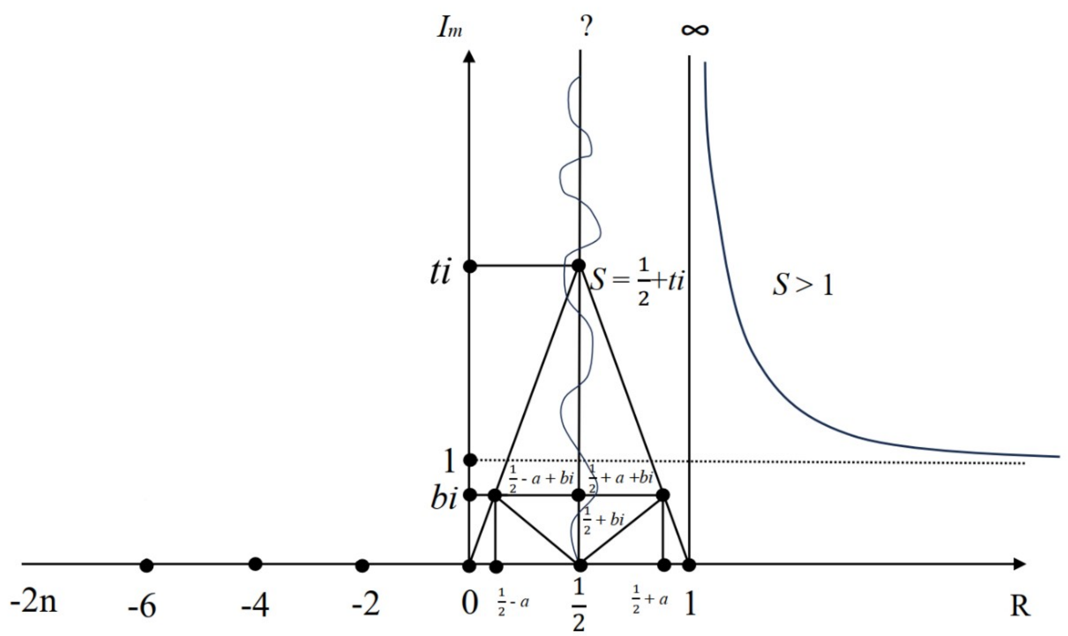

Figure 8: Riemann Hypothesis: All the Non-Trivial Zero Points of Riemann Zeta-Function are on the 1/2 Axis

$$s=\frac{1}{2}+ti\ \ t\in R$$

$$zp1=\frac{1}{2}-a+bi\ \ zp0=\frac{1}{2}+bi\ \ zp2=\frac{1}{2}+a+bi$$

$$zp1+zp2=\left(\frac{1}{2}-a+bi\right)+\left(\frac{1}{2}+a+bi\right)=1+2bi$$

$$zp2-zp1=\left(\frac{1}{2}+a+bi\right)-\left(\frac{1}{2}-a+bi\right)=2a$$

$$a, b\in R\ \ 0\le a\le\frac{1}{2}$$

As the Figure 9. If we have zero points of $\xi(s)$ on line-1/2 ± a as

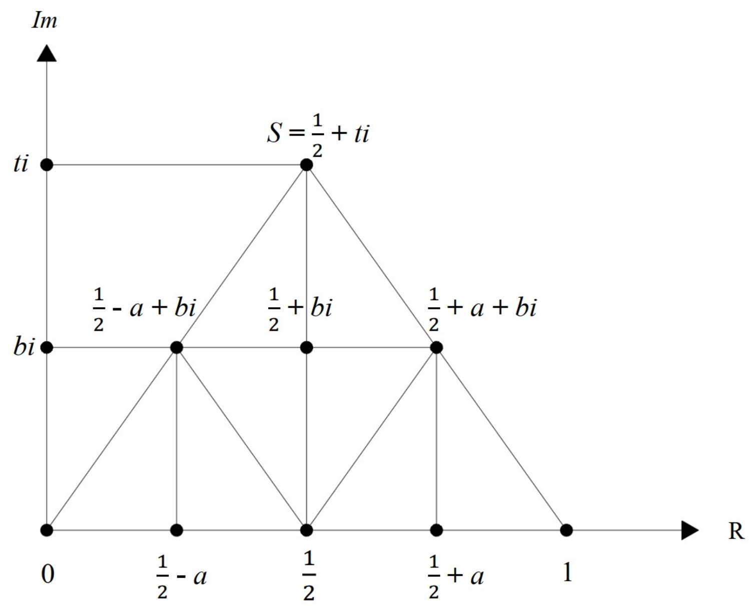

Figure 9: A Symmetry Structure about Line1/2+/-a at the Zero Piont *s* = 1/2 + *ti*

$$zp1 = \frac{1}{2} - a + bi \ \ zp2 = \frac{1}{2} + a + bi$$

And $s = \frac{1}{2} + ti \ \ t \in R$ is the first zero point on line-1/2

So

$$\xi\left(\frac{1}{2} - a + bi\right) = 0$$

$$\xi\left(\frac{1}{2} + a + bi\right) = 0$$

$$\xi\left(\frac{1}{2} - a + bi\right) * \xi\left(\frac{1}{2} + a + bi\right) = \sum_{n=1} \frac{1}{n^{\left[\left(\frac{1}{2} - a + bi\right) + \left(\frac{1}{2} + a + bi\right)\right]}} = \xi^2\left(\frac{1}{2} + bi\right) = 0$$

$$\xi\left(\frac{1}{2} + bi\right) = 0$$

So we can get a zero point of $\xi(s)$ as

$$zp0 = \frac{1}{2} + bi \ \ 0 < b < t \ \ b, t \in R$$

It is contrary to that $s = \frac{1}{2} + ti \ \ t \in R$ is the first zero point on line-1/2

As the Figure 10. If we have zero points of $\xi(s)$ on line $1/2 \pm a$ as

$$zp1 = \frac{1}{2} - a + bi \ \ zp2 = \frac{1}{2} + a + bi$$

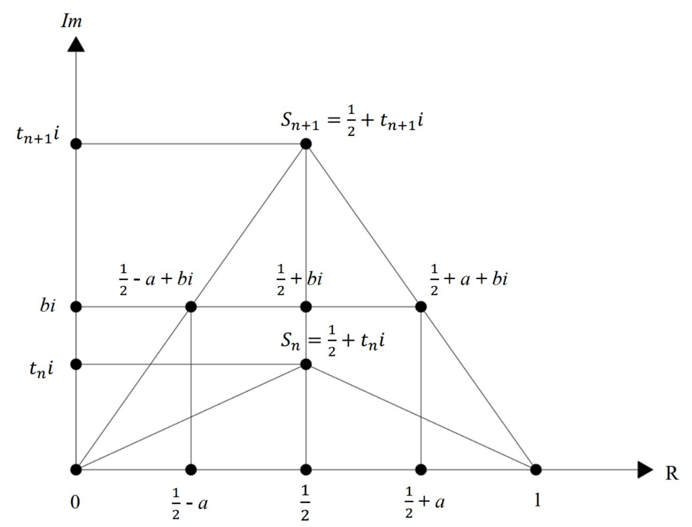

Figure 10: A Symmetry Structure about Line 1/2+/-*a* at the Zero-Point $s_n = 1/2 + t_n i$ and $s_{n+1} = 1/2 + t_{n+1} i$

And $s_n = \dfrac{1}{2} + t_n i \ t \in R$ is the No. n zero point on line-1/2

$s_{n+1} = \dfrac{1}{2} + t_{n+1} i \ t \in R$ is the No. n+1 zero point on line-1/2

We can get a zero point of $\xi(s)$ between s_n and s_{n+1} on line-1/2 as

$$zp0 = \frac{1}{2} + bi \ \ t_n < b < t_{n+1} \ \ b, t \in R$$

It is contrary to that s_n and s_{n+1} are the adjacent zero points on line-1/2

So on complex plane, we can have the symmetry structure about the line-1/2 with $zp = 1/2 \pm a \left(0 \le a \le \dfrac{1}{2} a \in R\right)$ show as on Figure 11.

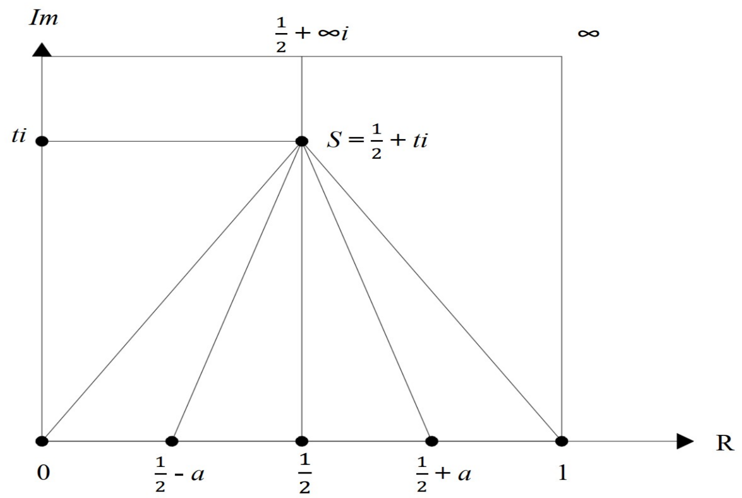

Figure 11: Symmetry Structure about the Line-1/2 with *zp* = 1/2 ± *a*

$$S = \frac{1}{2} + ti\left(t \in R\right)$$

$$zp = \frac{1}{2} \pm a\left(0 \le a \le \frac{1}{2} a \in R\right)$$

$$zp1 = \frac{1}{2} - a \;\; zp0 = \frac{1}{2} \;\; zp2 = \frac{1}{2} + a$$

$$zp1 + zp2 = \left(\frac{1}{2} - a\right) + \left(\frac{1}{2} + a\right) = 1$$

$$zp2 - zp1 = \left(\frac{1}{2} + a\right) - \left(\frac{1}{2} - a\right) = 2a$$

This is mean that there are no zero points on line-1/2 ± $a \left(0 \le a \le \frac{1}{2} \; a \in R\right)$

Hardy and Littlewood (1914) give a proof that there are infinite zero points on line-1/2.

So we give a proof that all the non-trivial Zero points of Riemann zeta-function are on the Line-1/2.

This is the proof of Riemann Hypothesis.

5. The Symmetry Number Structure about Line-1/2 Including All Numbers

In fact, we have a symmetrical number structure about line-1/2 as Figure 12.

And we can get a symmetrical number structure about line-1/2 as Figure 13. We should call it Reimann dynamic space.

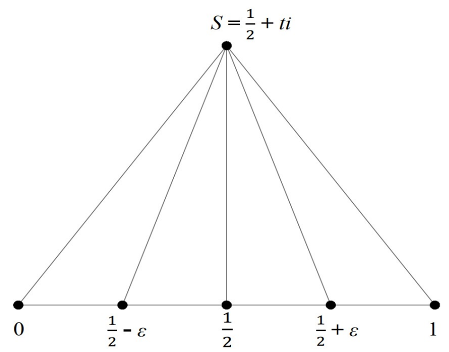

Figure 12: Symmetry Structure about the Line-1/2 with *zp* = 1/2 ± *ε*

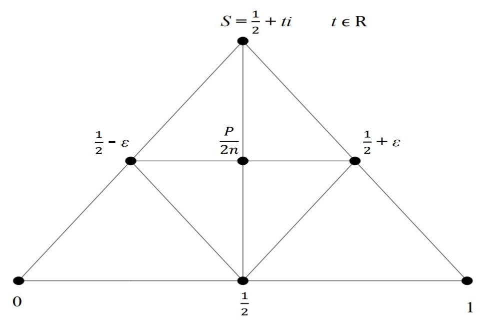

Figure 13: Reimann Dynamic Space

$1 + i^2 = 0$

$1 + 1/2\,(i + 1)\,(i - 1) = 0$

$$S = \frac{1}{2} + ti\left(t \in R\right)$$

$$zp = \frac{1}{2} \pm \varepsilon\left(\varepsilon = a + bi\ a, b \in R\ 0 \le a \le \frac{1}{2}\right)$$

$$\frac{P}{2n} = \begin{cases} \frac{1}{2^{N+1}} & n = 2^N P \\ \frac{3}{4} & n = 2\ P = 3 \\ 1 & n = 1\ P = 2 \end{cases}$$

$N \sim (0, 1, 2, 3, 4, ...)$ All natural numbers

$n \sim (1, 2, 3, 4, ...)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, ...)$ All prime numbers

We can have point $p1, p2, p3, p4$ and

$$p1 \in \left(\frac{1}{2} - a, \frac{p}{2n}\right)$$

$$p2 \in \left(\frac{p}{2n}, \frac{1}{2} + a\right)$$

$$p1 + p2 = \frac{p}{n}$$

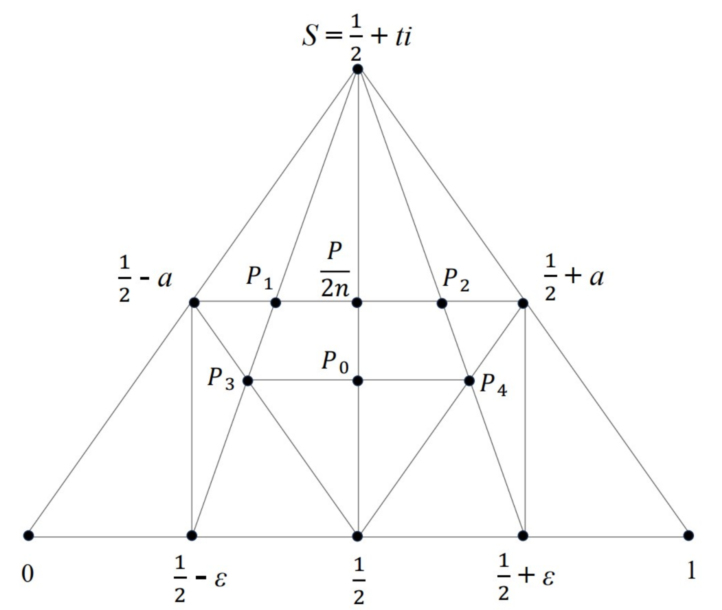

Figure 14: Reimann Dynamic Space with $p1$, $p2$, $p3$, $p4$

$$p3 \in \left(\frac{1}{2}-a, \frac{1}{2}\right)$$

$$p4 \in \left(\frac{1}{2}, \frac{1}{2}+a\right)$$

$p0 = 1/2(p3 + p4)$

And we can get Figure 15.

1. $zp = \frac{1}{2} + bi \ 0 < b < t \ b, t \in R$ (the proof of RH)

2. $\left(\frac{x}{z}\right)^n + \left(\frac{y}{z}\right)^n = 1$

$$\left(\frac{x}{z}\right)^n \leftrightarrow \frac{1}{2} - \frac{1}{2^n}$$

$\left(\frac{y}{z}\right)^n \leftrightarrow \frac{1}{2} + \frac{1}{2^n}$ (the proof of F.L.T)

3. $\frac{p0}{2n} \leftrightarrow \frac{1}{2} - \frac{1}{2n}$

$\frac{pn}{2n} \leftrightarrow \frac{1}{2} + \frac{1}{2n}$ (the proof of GC/BC/TPC)

$$\frac{pn \pm p0}{4n} \leftrightarrow \frac{1}{2}$$

4. $4n = 2n + 2n = [(n-1) + (3n+1)]$

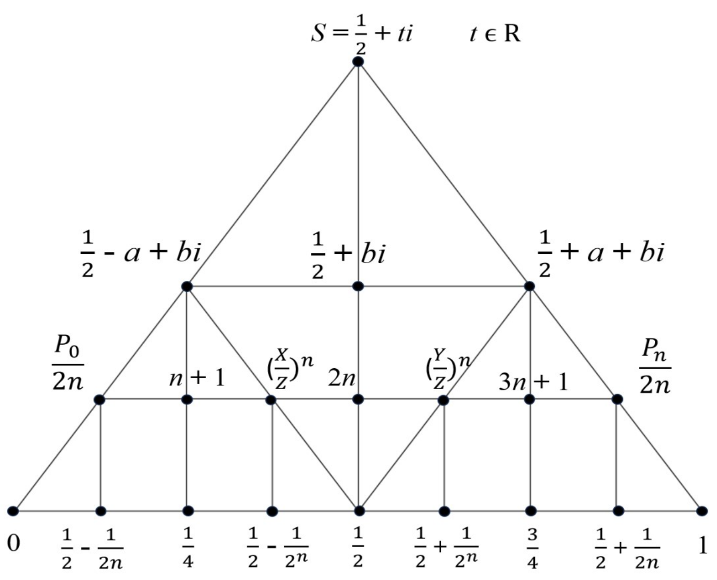

Figure 15: Reimann Dynamic Space and Number Conjectures

$$\lim_{n\to\infty}\left(\frac{n-1}{4n}\right)=1/4$$

$$\lim_{n\to\infty}\left(\frac{3n+1}{4n}\right)=3/4$$ (the proof of $3n+1$ conjecture)

And we have

$1/2 = 1/2,\ 0 = 1/2 - 1/2,\ 1 = 1/2 + 1/2$

$1 + (+i)^2 = 0$

$1 + 1/2(i+1)\,(i-1) = 0$

$\infty = 1+1+1+1+\ldots$

We called it $L^{1/2\pm\varepsilon}$ [0 1/2 1] and analytic continuation to $\begin{bmatrix}+\infty & -\infty\\ -\infty & +\infty\end{bmatrix}$ we can get Figure 16.

So we have:

$$1+\begin{bmatrix}+\infty & i & -\infty\\ 0 & 1/2 & 1\\ -\infty & -i & +\infty\end{bmatrix}\begin{bmatrix}1 & 1/2 & 0\\ \frac{1}{2}-\varepsilon & 1/2 & \frac{1}{2}+\varepsilon\\ 1 & 1/2 & 0\end{bmatrix}^{-1}=0$$

$$zp=\frac{1}{2}\pm\varepsilon$$

$$\varepsilon=a+bi\left(a,\ b\in R,\ 0\le a\le\frac{1}{2}\right)$$

$$zp1=\frac{1}{2}-\varepsilon,\ zp2=\frac{1}{2}+\varepsilon$$

We have

$$zp1+zp2=\left(\frac{1}{2}-\varepsilon\right)+\left(\frac{1}{2}+\varepsilon\right)=1$$

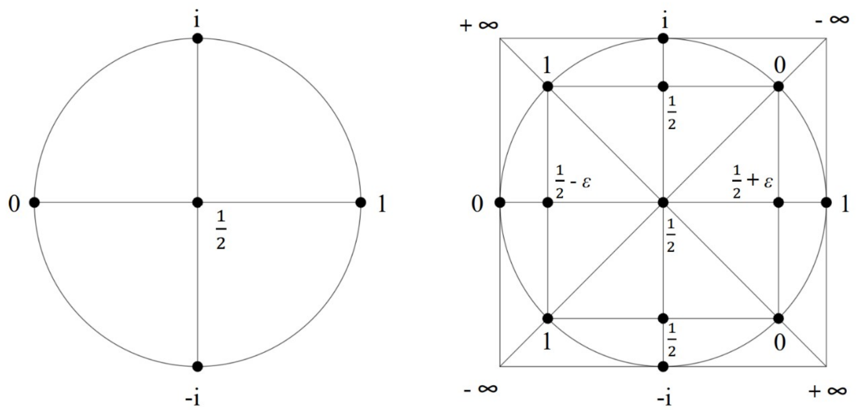

Figure 16: The Symmetry of $L^{1/2\pm\varepsilon}$[0 1/2 1] with $zp=\frac{1}{2}\pm\varepsilon$

$$zp2 - zp1 = \left(\frac{1}{2} + \varepsilon\right) - \left(\frac{1}{2} - \varepsilon\right) = 2\varepsilon = 2(a + bi)$$

And we have

$$n^2 = \frac{1}{2} \cdot n \cdot 2n = \sum^{N} \frac{1}{2} \sum^{N} \frac{1}{2^N} \left[\left(\frac{1}{2} - \varepsilon\right) + \left(\frac{1}{2} + \varepsilon\right)\right]$$

$N \sim (0, 1, 2, 3, 4, ...)$ All natural numbers

$n \sim (1, 2, 3, 4, ...)$ All natural numbers excepted 0

We can get a matrix (n x n)

$$\begin{bmatrix} 1/2 & \cdots & \frac{1}{2^n}(1/2+\varepsilon) \\ \cdots & 1/2 & \cdots \\ \frac{1}{2^n}(1/2-\varepsilon) & \cdots & 1/2 \end{bmatrix} (n \times n)$$

The $tr(A) = 1/2*n$

We have

$$0 = \frac{1}{2} - \frac{1}{2},\ 1 = \frac{1}{2} + \frac{1}{2},\ 2 = 1 + 1$$

$1 + i^2 = 0,\ 1/2 + i^{4N+1} = 1/2 + i$

$\infty = 1 + 1 + 1 + 1 + \cdots$

$p0 \in P < 2n, pn \in P > 2n$

$N \sim (0, 1, 2, 3, 4, ...)$ All natural numbers

$n \sim (1, 2, 3, 4, ...)$ All natural numbers excepted 0

$P \sim (2, 3, 5, 7, ...)$ All odd prime numbers

$$S = \frac{1}{2} + t\,(t \in R)$$

$$zp = \frac{1}{2} \pm \varepsilon\left(\varepsilon = a + bi,\ a,\ b \in R,\ 0 \le a \le \frac{1}{2}\right)$$

And we find that

1. $1 + e^{\pi i} = 0$ (Eulaer's Formula)

$$1 + i^2 = 0,\ 1 + \frac{1}{2}(i+1)(i-1) = 0,\ (1+i)(1-i) = \sum \frac{1}{2^N}$$

$$1 + e^{\pi i} = 0,\ 1 + \frac{1}{2}\left(e^{ip\pi} - e^{i2N\pi}\right) = 0$$

$N \sim (0, 1, 2, 3, 4, ...)$ All natural numbers

$p \sim (3, 5, 7, ...)$ All odd prime numbers

2. $2(n + 1) = pn + p0$

$pn - 2n + p0 = 2$

And

$2n - pn + p0 = 2$

It is like the Euler's Polyhedron Formula

We can get Figure 17. This is a symmetry number structure about line-1/2 including all numbers and the equivalence structure of $S^{\infty} + i$ is shown as Figure 18.

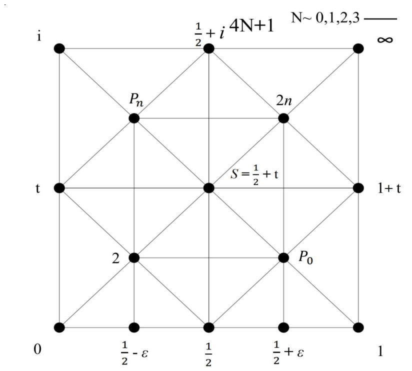

Figure 17: The Symmetry of $S^{\infty} + i$

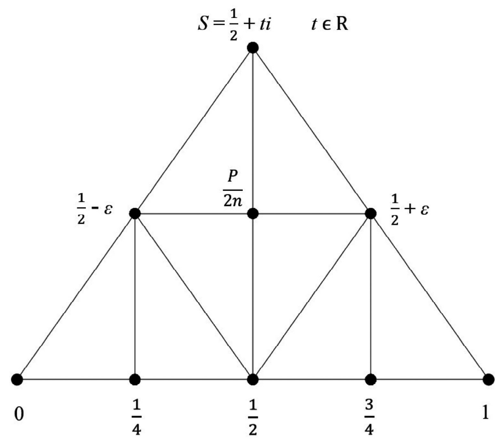

Figure 18: The Equivalence Structure of $S^{\infty} + i$

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