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Ismail A Mageed's Megaannum Theory of Fractal Topology: Open Problems, Applications, and Outlook for Tomorrow

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Abstract

In both mathematics and the real world, fractal geometry—which Benoît Mandelbrot first introduced as a formal mathematical idea—has radically changed how we view complexity, irregularity, and scale. Beyond the breathtaking images of sets like the Mandelbrot set is a rich and complicated subject known as “fractal topology,” which this article defines as the investigation of the topological features of fractal sets. These characteristics, including connectedness, compactness, and dimensional disparity, go against what we know about Euclidean geometry and give us a new way to talk about things that happen from the smallest parts of neurons to the way galaxies are spread out in the universe. Starting with its basic concepts, including the crucial difference between topological and Hausdorff dimension, this paper offers a thorough survey of fractal topology. It then investigates major unsolved questions including the Mandelbrot Local Connectedness (MLC) assumption and the difficulty of developing a coherent calculus on fractal spaces, therefore navigating the bounds of present study. Later, the study examines the broad and expanding scene of uses, highlighting the usefulness of fractal topology in areas as varied as finance, computer graphics, medicine, and telecommunications. It ultimately provides a forward-looking view of the discipline’s future, forecasting its fusion with artificial intelligence, possible part in creating new metamaterials, and continuing development as a basic instrument for grasping complexity. This investigation shows that fractal topology is a crucial and growing field of current science and mathematics rather than only a little hobby.

Keywords: *Fractal geometry, Fractal topology, Hausdorff dimension, Open problems, Mandelbrot set, Complex dynamics, Fractal applications*

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1. Introduction

Mathematics was ruled for millennia by the smooth, flawless shapes of Euclidean geometry—lines, circles, and polyhedra. Less symmetric tangled toroidal polyhedra are similar to torus knots (Hyde and Evans, 2022), which are created by wrapping a loop around the surface of a torus, or donut shape. Examples of these knots include the trefoil knot and the Hopf link (Hyde and Evans, 2022), which are labelled based on the minimum number of crossings needed to draw them.

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The process involves winding strands around the torus in a specific way, creating a structure that can be visualized as a “two-track railway” that twists and intertwines (Hyde and Evans, 2022), resulting in different knot types depending on the number of twists applied. In the context of the article, torus knots are specific types of knots that can be visualized on a torus, which is a doughnut-shaped surface. They are labeled using “railway labels” that indicate their structure; for example, the $\frac{2}{2}$ knot is known as the Hopf knot, and the $\frac{3}{2}$ knot is called a trefoil knot. These labels help identify the knots based on how many times they loop around the torus and how they intertwine with themselves. The illustrations are visually displayed in Figure 1 (Hyde and Evans, 2022).

A tangled polyhedron (Hyde and Evans, 2022), like the one described as $\left(\frac{2}{3}\right)^3 \theta$, is created by wrapping three tracks around a special surface called a genus-2 bitorus, which has two holes. Each edge of this polyhedron winds around these tracks (Hyde and Evans, 2022), creating twists that result in knots, such as the trefoil knot. This means that the edges are intertwined in a way that they cannot be straightened out without crossing over each other (Hyde and Evans, 2022), making the structure uniquely tangled and chiral, meaning it has a specific “handedness” or orientation.

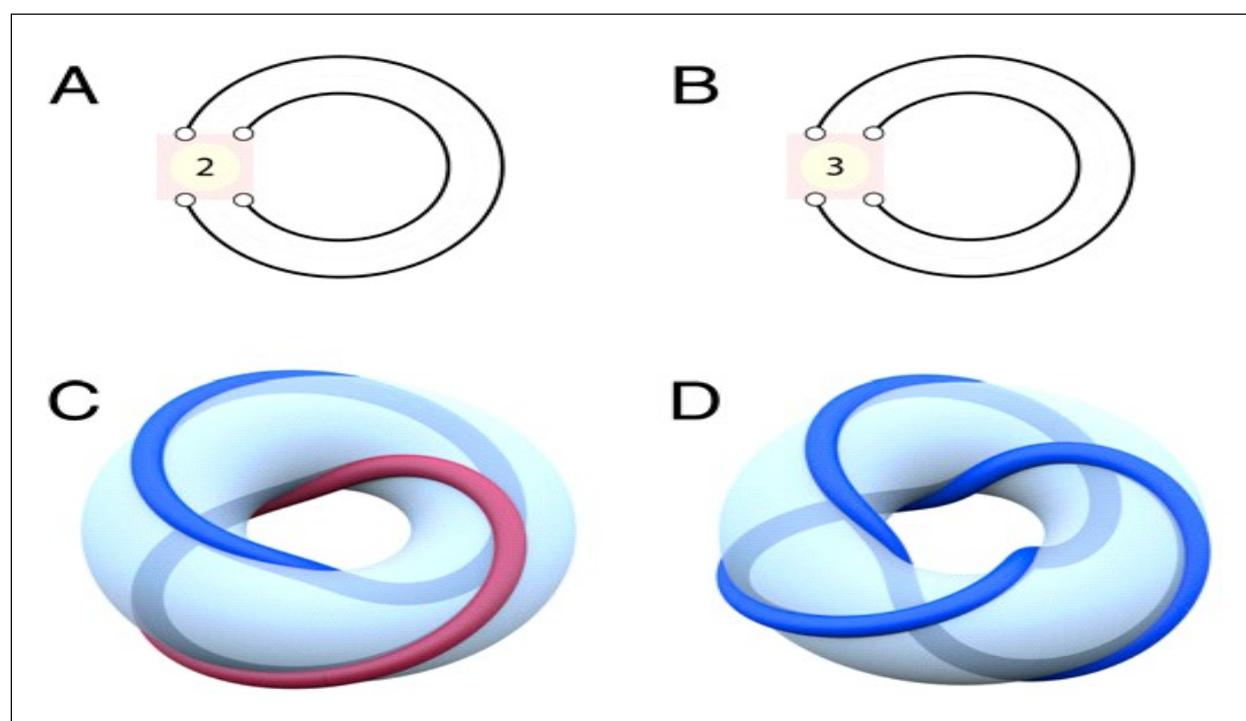


Figure 1: The Visuals Describe Two Types of Railways that are Arranged in a Circular Shape Called a Torus, where Each Railway Consists of Two Parallel Strands. In the First Example (A), the Railway has Two Half-Twists, While in the Second Example (B), it has Three Half-Twists, Creating Different Types of Tangles in Three-Dimensional Space. Additionally, it Mentions Specific Knot Structures, Like the Hopf link $\left(\frac{2}{2}; C\right)$ and the Trefoil Knot $\left(\frac{3}{2}; D\right)$, Which are Examples of How Strands Can Intertwine in Complex Ways

The construction of the $\left(\frac{2}{3}\right)^3 \theta$ tangled polyhedron involves creating a structure where three strands twist around each edge of a special graph, forming a triple-helix shape, as depicted in Figure 2 (Hyde and Evans, 2022). This design allows the polyhedron to maintain a high level of symmetry while being tangled, meaning its edges are not straight but rather curved or kinked. The result is a visually complex shape that showcases how entangled structures can be formed from basic geometric principles, making it relevant for understanding both theoretical and practical applications in chemistry and materials science.

Tangled polyhedra are complex three-dimensional shapes that consist of multiple interconnected loops and structures. In this specific case (see Figure 3 (c.f., Hyde and Evans, 2022)), the polyhedron has four different parts, each colored red, blue, green, and black. The red, green, and blue components are simple loops, while the black component is a more complex structure called a θ -graph, which branches out, creating a unique entangled form.

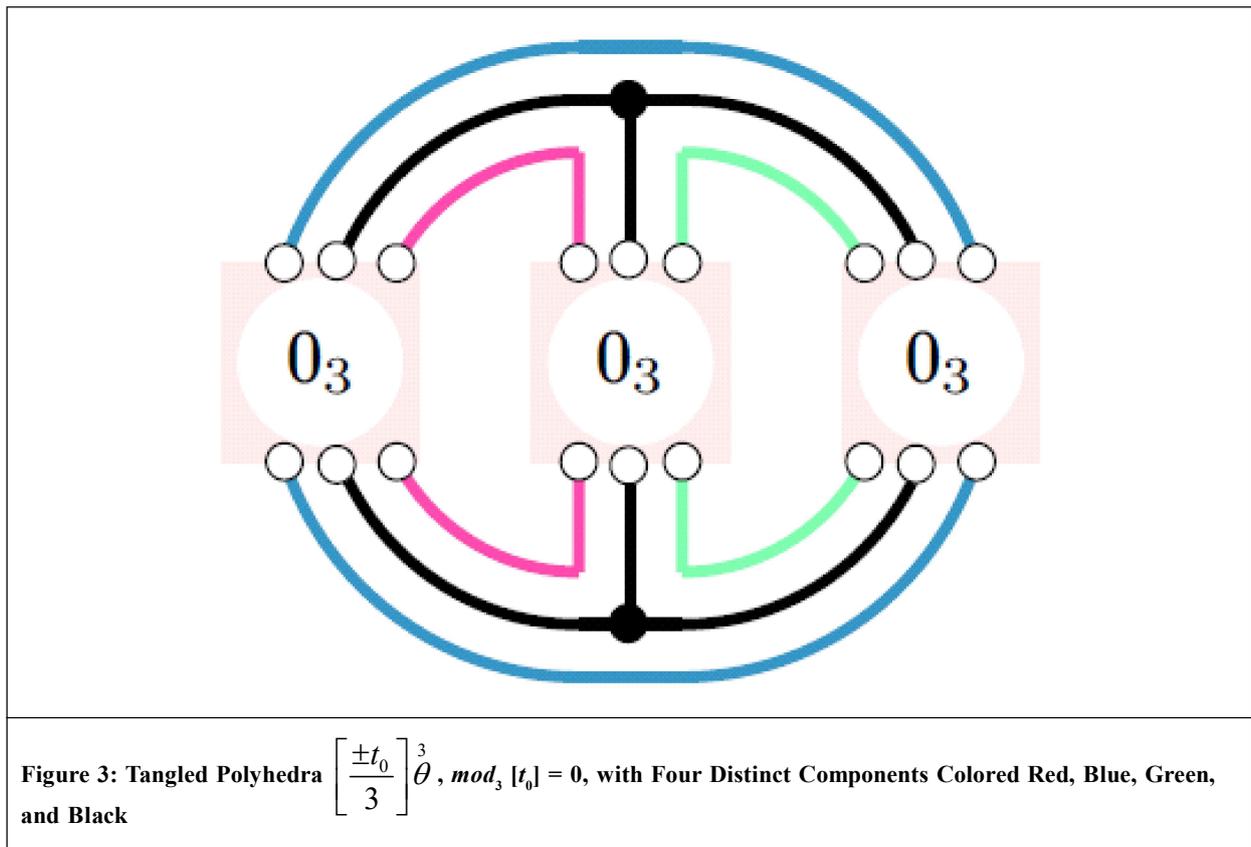
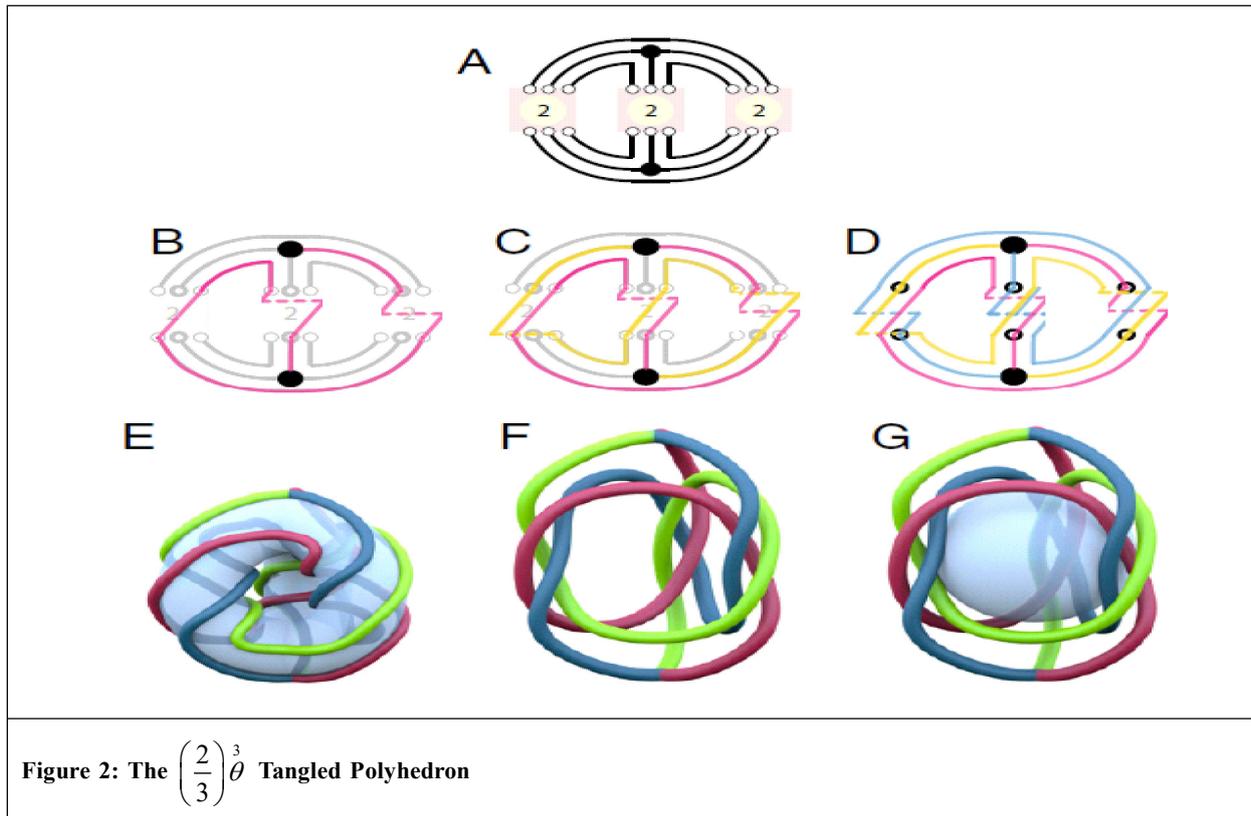


Figure 4 (Hyde and Evans, 2022) describes different types of tangled Platonic polyhedra, which are three-dimensional shapes that have symmetrical properties similar to traditional Platonic solids but are constructed using strands that create twists and turns. For example (Hyde and Evans, 2022), the chiral tangled tetrahedron and octahedron are made from three and five strands, respectively, while the tangled cubes and icosahedrons have more complex structures.

These tangled shapes maintain the same vertices as their untangled counterparts, but their edges are curved or kinked, allowing for unique entangled forms that are still symmetrical.

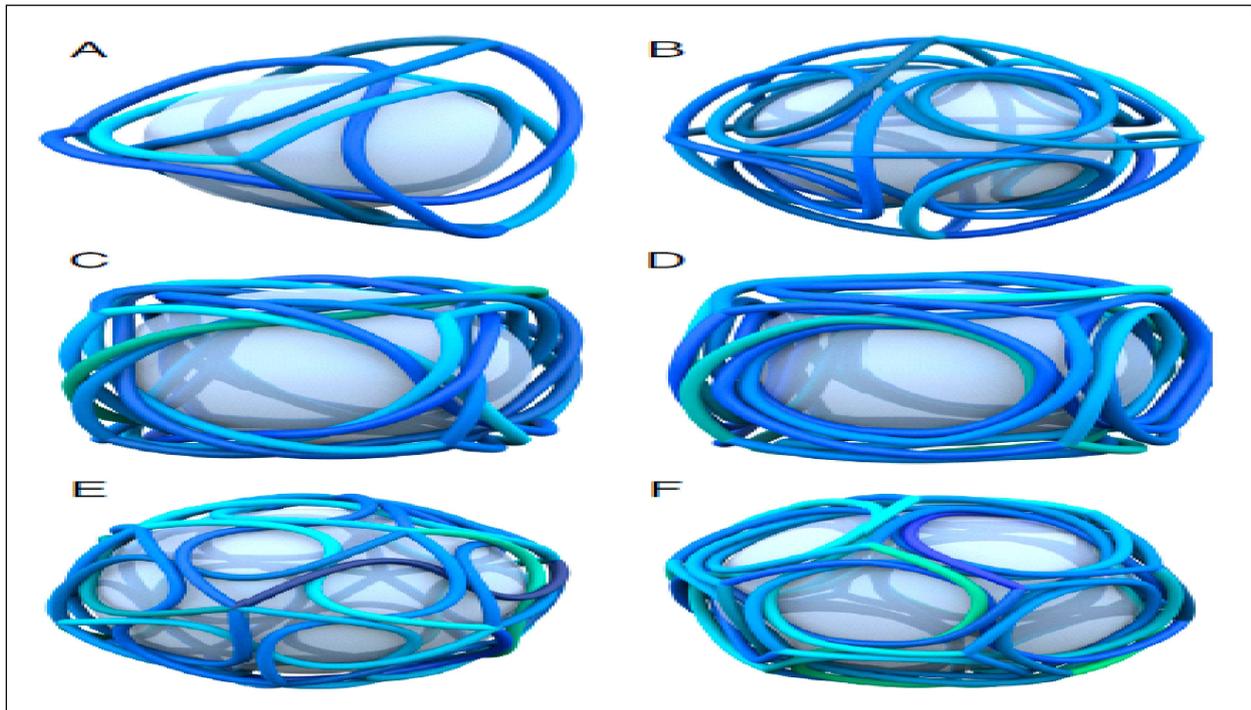


Figure 4: Simple Platonic Polyhedra with Tangled Vertex, Edge, and Face-Transitive Helices Composed of Three, Five, or Seven Strands

Figure 4 refers to specific types of tangled polyhedra that have been mathematically represented in a way that shows how they can be arranged without overlapping edges. The numbers in brackets indicate the structure and complexity of these tangles, and Figure 5 (Hyde and Evans, 2022), visually illustrates these arrangements, helping to understand their geometric properties. Essentially, this part of the text highlights the visual representation of complex shapes that are derived from the study of tangled polyhedra.

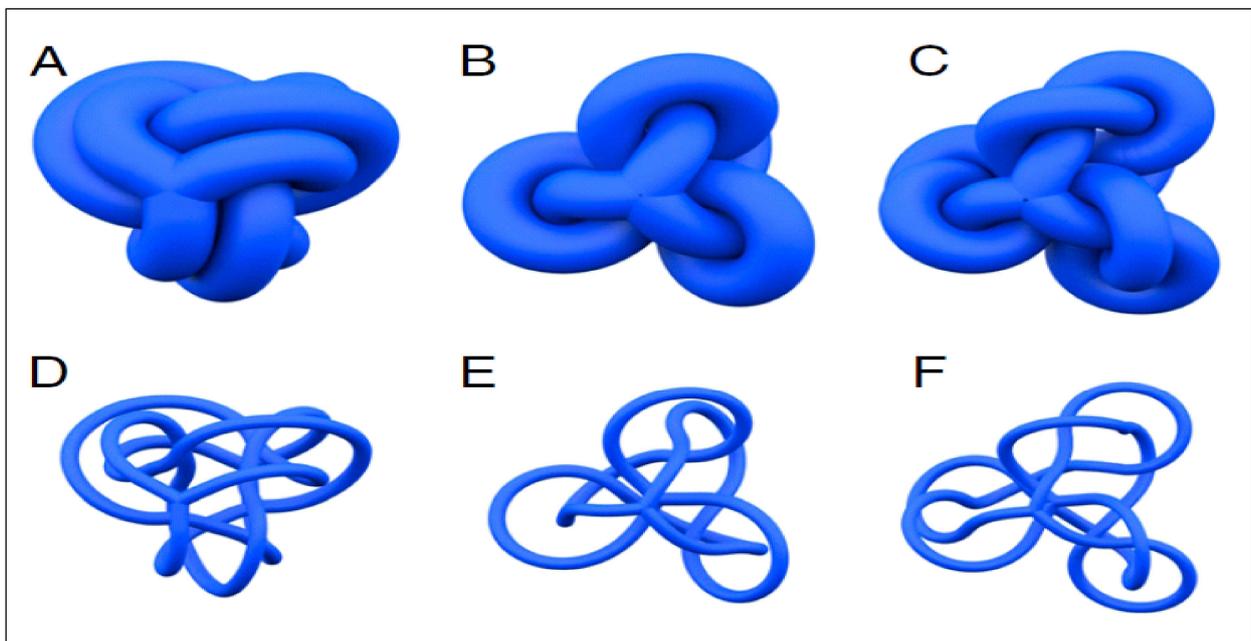


Figure 5: Numerical Realizations of Tight Embeddings of the $\left(\frac{2}{3}\right)^3\theta$, $\left(\frac{1}{5}\right)^3\theta$, and $\left(\frac{1}{7}\right)^3\theta$ Tangles

Certain complex shapes called “tangled polyhedra” can be created using helical windings, which are very symmetric but not as symmetric as traditional Platonic polyhedra, as depicted in Figure 6 (Hyde and Evans, 2022). To show that these tangled shapes cannot have the same level of symmetry as the untangled ones, the authors analyze a specific type of polyhedron called the θ -polyhedron. They demonstrate that if this shape has a certain symmetry, its edges must lie flat in specific planes, allowing it to be drawn without any crossings, which means it is untangled.

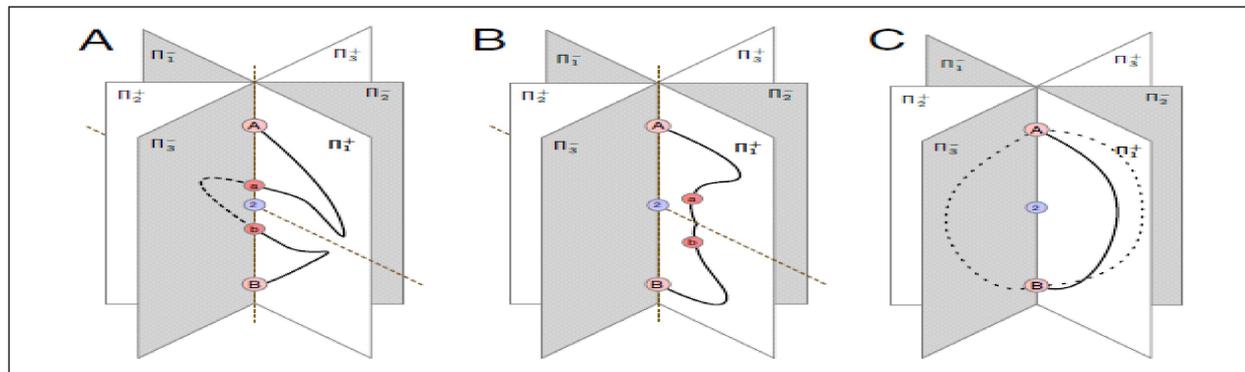


Figure 6: Labelled $\Pi_1, \Pi_2,$ and Π_3 are the Mirror Planes of a θ -Polyhedron with Symmetry $*223$, Each Divided into Half-Planes Π_i^+ and Π_i^-

Describing human-made items, these forms sufficed but fell short in recording the complex and erratic patterns present in nature. As Benoît Mandelbrot (Mageed and Bhat, 2022; Mageed, 2023a; Mageed, 2023b; Mageed and Bhat, 2023; Mageed and Mohamed, 2023; Mageed and Bhat, 2024a; Mageed and Bhat, 2024b; Mageed, 2024a-j; Mageed and Li, 2025; Mageed and Mohaisen, 2025; Mageed, 2025a-t) famously asked, “Is a mountain a cone? Is a coastline a circle?” His answer was a resounding no, proposing a new geometry of nature based on the concept of the “fractal” (Mageed and Bhat, 2022; Mageed, 2023a; Mageed, 2023b; Mageed and Bhat, 2023; Mageed and Mohamed, 2023; Mageed and Bhat, 2024a; Mageed and Bhat, 2024b; Mageed, 2024a-j; Mageed and Li, 2025; Mageed and Mohaisen, 2025; Mageed and Ahmed, 2025; Mageed, 2025a-t). A fractal is, broadly speaking, a geometric object that is self-similar, exhibiting repeating patterns at every scale, and possesses a fractal dimension that exceeds its topological dimension (Fraser, 2020).

This paper focuses on what can be termed “fractal topology,” which we define as the subfield concerned with the topological properties of these complex sets. Although fractal geometry typically stresses metric aspects like the Hausdorff dimension, fractal topology looks at features including connectedness, compactness, path-connectedness, and the shape of boundaries (Akhmet *et al.*, 2020). The Cantor set, for instance, is entirely separated; the Koch Snowflake has an endless length yet linked border; the Sierpinski (2020) carpet is a linked set whose removal splits the plane (Wu *et al.*, 2020).

In the late twentieth century, Benoît Mandelbrot (Mageed, 2025u) revolutionised the intriguing world of fractals, which are objects that exhibit self-similarity across different scales. One of the most stunning examples in this category

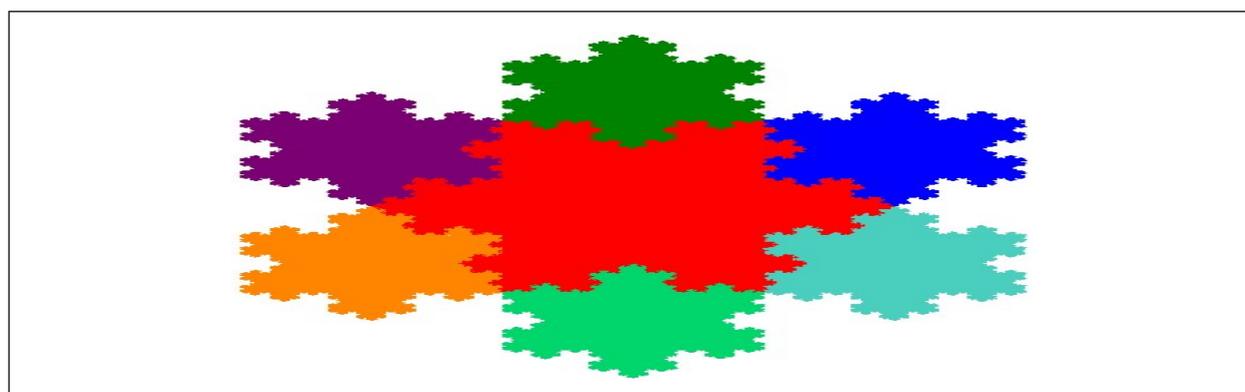


Figure 7: The Elegant Koch Snowflake

is the Koch snowflake, as depicted in Figure 7 (Husain *et al.*, 2022a) and Figure 8 (c.f., Peitgen *et al.*, 2004), a wonderful work of grace and clarity. The amazing geometry of Koch Snowflake is manifested through these figures.

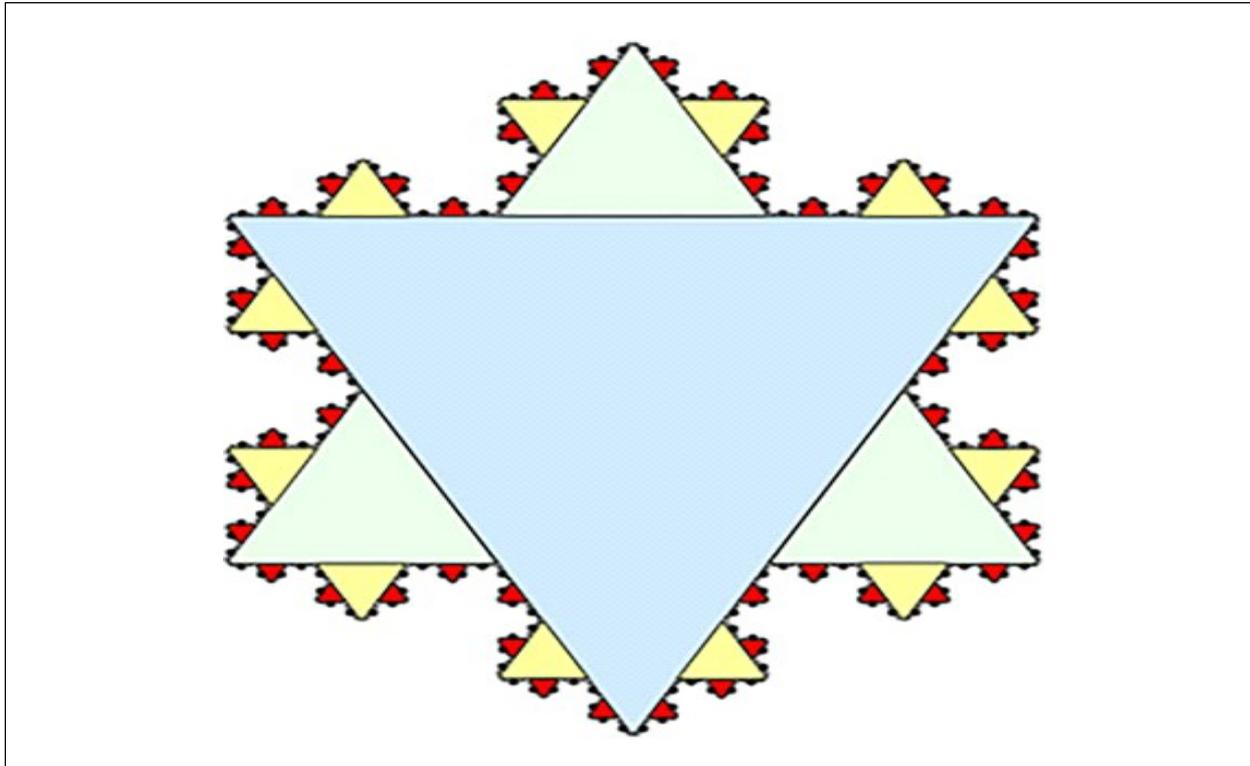


Figure 8: The Magnificence of Koch Snowflake Fractal

It all begins with an equilateral triangle known as the initiator. In each stage of the process, or iteration, the middle third of each line segment is changed by two sides of a smaller equilateral triangle pointing outward—this is known as the generator.

And this process goes on forever (von Koch, 1904; Aravindraj *et al.*, 2023). The visual illustration is displayed by Figure 9 (c.f., Aravindraj *et al.*, 2023).

What's truly remarkable about the Koch Snowflake is that it has two well-known properties. First, its perimeter is infinite. The boundary length rises by a factor of $\frac{4}{3}$ with each iteration, resulting in an ever-expanding perimeter as the iterations continue indefinitely. Second, despite its infinite perimeter, the area is finite, eventually reducing to $(\frac{8}{5})$ the area of the original triangle (Akhmet *et al.*, 2020). This intriguing mix of finite and infinite measurements within a single

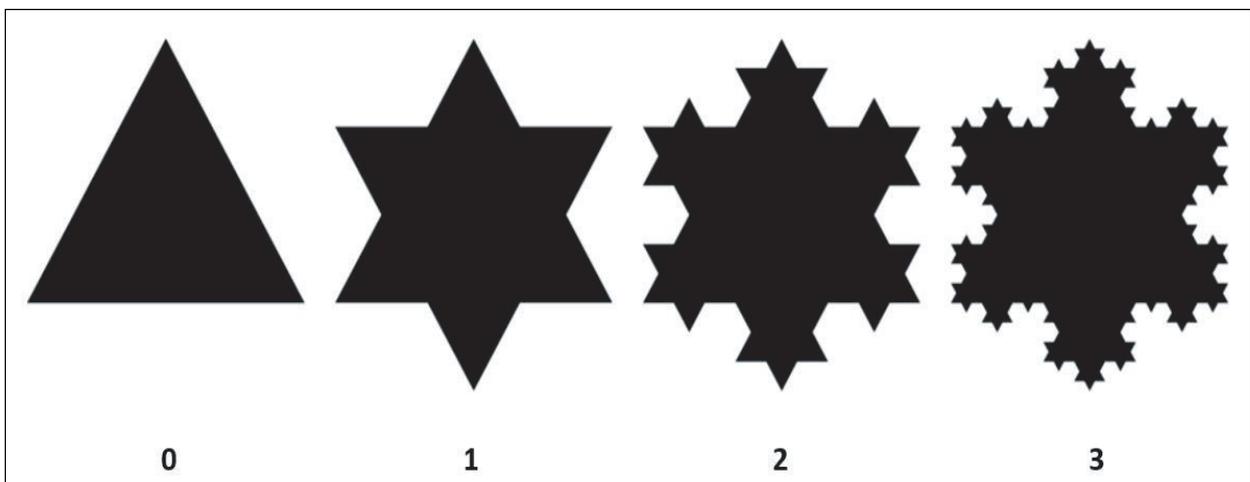


Figure 9: Expanding Koch Snowflake Fractal Pattern Layers

geometric shape is a fundamental aspect of fractal geometry. Additionally, the Koch curve is continuous everywhere but differentiable nowhere, which posed a significant challenge to traditional calculus (Husain *et al.*, 2022a, 2022b). It has a Hausdorff dimension of $\log(4)/\log(3) \approx 1.2618$, which indicates its “roughness” or ability to fill space. This means that is more than simply a one-dimensional line but not nearly a two-dimensional plane (Bunimovich and Skums, 2024). While these qualities are well understood, they pave the way for even more profound, unresolved concerns.

Learning these qualities is essential for both theoretical progress and real-world application rather than just a scholarly pursuit.

There are deep and challenging questions in the study of fractal topology that push the boundaries of what we know about math today. These open problems, particularly in the realm of complex dynamics, attract intense research interest (Strogatz, 2024). Concurrently, the application of fractal concepts has permeated nearly every scientific and engineering discipline. From designing more efficient antennas (Karmakar, 2021) and compressing digital images (Bremer and Wu, 2021) to modeling the growth of tumors (West, 2021) and the volatility of financial markets (Mageed and Bhat, 2022; Mageed, 2023a; Mageed, 2023b; Mageed and Bhat, 2023; Mageed and Mohamed, 2023; Mageed and Bhat, 2024a; Mageed and Bhat, 2024b; Mageed, 2024a-j; Mageed and Li, 2025; Mageed and Mohaisen, 2025; Mageed, 2025a-t), fractals provide an indispensable framework for analyzing and simulating complexity.

This paper will first establish the foundational concepts of fractal topology, highlighting the crucial role of dimension. It will then delve into some of the most significant open problems that define the field’s research frontier. Afterwards, a look at the many applications of fractal topology will also be shown. The essay will end with a look ahead, looking at how fractal topology, new ways of thinking about computers like artificial intelligence (Mageed *et al.*, 2024a-c), and new areas of science could all work together.

2. The Topological Landscape of Fractals

The defining characteristic of a fractal set is the discord between our intuitive sense of its dimension and its measured complexity. This is formalized by the distinction between its topological dimension (d_T) and its fractal dimension, most commonly the Hausdorff dimension (d_H).

The topological dimension is an integer that corresponds to the classical notion of dimensionality. A point has $d_T = 0$, a line has $d_T = 1$, a surface has $d_T = 2$, and so on (Carrington, 2024). It is invariant under homeomorphisms—the continuous deformations that define topological equivalence. A key feature of fractals is that they often possess a non-integer Hausdorff dimension which is strictly greater than their topological dimension (Fraser, 2020).

The Hausdorff dimension, d_H , was introduced by (Hausdorff, 1918) and provides a way to measure the “metric size” of arbitrarily complex sets. It is calculated by considering how the number of small sets of diameter δ needed to cover the set, $N(\epsilon)$, scales as $\epsilon \rightarrow 0$. If $N(\epsilon) \propto \epsilon^{-D}$, then D is the fractal dimension. For a simple line segment, $D = 1$. For a square, $D = 2$. However, for the middle-thirds Cantor set, constructed by recursively removing the middle third of line segments, one finds $d_T = 0$ (it consists of disconnected points) but its Hausdorff dimension is $d_H = \log(2)/\log(3) \approx 0.631$ (Barnsley, 2012). This fractional value quantifies the set’s complexity and space-filling capacity, which is more than a collection of points but less than a continuous line.

Other key topological properties include:

- **Connectedness:** This property is surprisingly varied in fractals. The Cantor set is totally-disconnected (Sierpinski, 2020). In contrast, the Koch curve and Sierpinski gasket are connected, and even path-connected (Burrell *et al.*, 2021). The connectedness of Julia sets in complex dynamics is a central topic of study (Beardon, 1991).
- **Compactness:** Most classic fractals generated by Iterated Function Systems (IFSs), like the Sierpinski triangle or the Koch snowflake, are compact sets (Bárány *et al.*, 2023). Several theoretical findings (Moran, 2019) and the existence of attractors for the dynamical systems producing them depend on this feature, which combines being closed and bounded in Euclidean space.
- **Self-Similarity:** A set F is considered strictly as self-similar if it is made up of smaller copies of itself, each scaled by the same amount (Lapidus and Radunovic, 2024). Although many classic fractals have this characteristic, others, like the Mandelbrot set, display a more complex kind known as quasi-self-similarity, in which at varied sizes distorted,

recognizable copies of the set show (Leclerc, 2023). Many uses, including fractal image compression (Ahmed *et al.*, 2020), rely on this trait.

Understanding this interplay between topological properties and metric complexity is the core of fractal topology. It provides the necessary language to move from simple observation to rigorous analysis.

3. Frontiers of Ignorance: Open Problems in Fractal Topology

Despite decades of progress, fractal topology still has several deep, unanswered questions. Often, these unresolved issues arise at the intersection of dynamical systems with analysis and topology.

3.1. The MLC Conjecture and the Mandelbrot Set

Among fractal geometry's most well-known items is the Mandelbrot set, M . In the complex plane, it is the collection of all parameters c for which the orbit of $z_0 = 0$ under the

$$z_{n+1} = (z_n)^2 + c$$

remains bounded (Douady, 1982). Its boundary is extraordinarily complex and is conjectured to have a Hausdorff dimension of 2 (Duminil-Copin, 2022).

The most significant open problem concerning M is the Mandelbrot Local Connectivity (MLC) conjecture. It posits that the Mandelbrot set is locally connected (Clark and van Strien, 2023). A space is locally connected if for every point, any neighbourhood of that point contains a smaller, connected neighbourhood. While this sounds like a simple technical property, its proof would have profound implications. Jean-Christophe Yoccoz made significant progress, proving that M is locally connected at all parameters c that are not "infinitely renormalizable" (Banaji and Kolossváry, 2021). However, a full proof remains elusive. If MLC is true, it would imply that every hyperbolic component of M 's interior corresponds to a specific type of periodic behavior, effectively making the Mandelbrot set a complete "parameter map" for the dynamics of quadratic polynomials (Milnor, 2000). This would solidify its role as a universal "dictionary" for complex dynamics.

3.2. Dimension of Fractal Boundaries and Measures

It is possible to calculate the Hausdorff dimension for many self-similar sets, but determining its value for more complex fractals, such as those caused by nonlinear dynamical systems or random processes, can be challenging. A major class of open problems involves finding the dimension of basin boundaries. For instance, in the Newton-Raphson method applied to polynomials in the complex plane, the boundaries separating the basins of attraction for different roots are often fractals (Shemyakov *et al.*, 2020). Determining the precise Hausdorff dimension of these boundaries is generally an unsolved problem (Das *et al.*, 2021).

Similarly, the multifractal formalism, which describes sets where the fractal dimension varies locally, presents its own challenges (Jaerisch and Sumi, 2020). Validating the multifractal spectrum for measures arising from physical processes like turbulence remains a major research area, connecting fractal topology directly with statistical physics (Kwapień *et al.*, 2023).

3.3. Analysis on Fractals

Classical calculus, based on differentiation and integration, relies on the smooth, locally linear nature of Euclidean space. These tools break down on fractals, which lack well-defined tangents and have infinite length between any two points. A major ongoing effort is the development of a consistent theory of "analysis on fractals."

Significant progress has been made by defining Laplacians on fractals using two main approaches: one based on probabilistic methods (Brownian motion on fractals) and another on analytic methods using resistance networks (Bárány *et al.*, 2023). These "fractal Laplacians" lead to differential equations, spectral theory, and harmonic analysis on fractals (Golmankhaneh and Tunç, 2020). However, a comprehensive theory is far from complete. Open questions include understanding the full spectrum of these Laplacians, developing a corresponding theory of vector calculus (e.g., defining divergence and curl), and applying this new calculus to physical models like wave propagation or heat flow in fractal media (Bernstein, 2021). The resolution of these problems would represent a paradigm shift, creating a mathematical toolkit as suited to irregularity as classical calculus is to smoothness.

4. The Fractal in Practice: A Survey of Applications

The abstract concepts of fractal topology have found remarkably concrete applications across science and technology. The principle that “nature is fractal” has become a powerful modeling tool.

- **Physics and Engineering:** The concept of fractal antennas, pioneered by (Karmakar, 2021) revolutionized antenna design. By using fractal shapes like the Sierpinski carpet, engineers can create compact antennas that are efficient (Anguera *et al.*, 2020). In statistical physics, percolation theory uses fractal concepts to model the connectivity of random systems, with applications to fluid flow in porous media and the behavior of disordered materials (Stauffer and Sahimi, 2021).
- **Computer Science and Graphics:** The most prominent application is fractal image compression. Michael Barnsley showed that any image could be approximated by the attractor of an Iterated Function System (IFS), allowing for massive compression ratios by storing the IFS codes instead of the pixel data (Gupta *et al.*, 2020). It is possible to calculate the Hausdorff dimension for many self-similar sets, but determining its value for more complex fractals, such as those caused by nonlinear dynamical systems or random processes, can be challenging. Fractals are indispensable in computer graphics for the procedural generation of realistic natural landscapes including mountains, beaches, clouds, and flora, therefore freeing artists from having to manually recreate every element (Yu *et al.*, 2024).
- **Biology and Medicine:** The human body has a lot of fractal structures that are made to be efficient. The lungs (trachea to alveoli), the circulatory system, and the nervous system all show fractal scaling to make the surface area as big as possible for exchange or to make the volume as big as possible for distribution in a small space (West, 2021). Diagnostics makes use of this understanding. For instance, a strong prognostic indicator is the fractal dimension of a tumour’s boundary seen in a biopsy or medical image; a more irregular, higher-dimensional boundary frequently denotes greater malignancy and aggressiveness (Kis and Güleriyüz, 2020).
- **Finance and Economics:** Early research by Benoît Mandelbrot on cotton price swings revealed that they did not follow a normal distribution but instead displayed erratic, unpredictable swings with fractal properties (Mageed and Bhat, 2022; Mageed, 2023a; Mageed, 2023b; Mageed and Bhat, 2023; Mageed and Mohamed, 2023; Mageed and Bhat, 2024a; Mageed and Bhat, 2024b; Mageed, 2024a-j; Mageed and Li, 2025; Mageed and Mohaisen, 2025; Mageed, 2025a-t). This paved the path for fractal models of financial markets, which try to catch things like volatility clustering and “long memory” effects that are not seen in normal models (Blackledge and Lamphiere, 2021). While still debated, this approach offers a richer framework for risk management (Li *et al.*, 2020).

5. The Shape of Things to Come: Outlook for Tomorrow

Fractal topology will likely be defined in the future by growing interaction with other areas and the creation of more sophisticated theoretical and computing capabilities.

- **Synergy with Artificial Intelligence (AI):** Together, machine learning and fractal analysis show great potential. AI algorithms could be trained to recognize fractal patterns in complex datasets, potentially leading to breakthroughs in medical diagnostics (e.g., automatically classifying tumor malignancy from images), material science (e.g., identifying stress fractures), and seismology (e.g., predicting earthquakes from fractal patterns in micro-tremors) (Kubo *et al.*, 2024). Conversely, fractal principles could inspire new neural network architectures that better handle hierarchical, multi-scale data (El-Kenawy *et al.*, 2021).
- **New Physical and Material Applications:** The development of metamaterials—engineered materials with properties not found in nature—is a burgeoning field where fractal topology is poised to play a key role. Fractal-based designs could be used to create materials with unique electromagnetic properties, such as perfect absorbers or cloaking devices operating over broad frequency bands (Zhang *et al.*, 2023). Similarly, fractal geometry can inform the design of more efficient catalysts, batteries, and heat exchangers by maximizing reactive surface areas (Chauhan *et al.*, 2023).
- **Deepening Theoretical Foundations:** The quest for a comprehensive theory of analysis on fractals will continue, potentially leading to a “fractal calculus” that enables the modeling of physical laws on irregular domains. This could revolutionize our understanding of quantum gravity, where some theories posit a fractal structure for spacetime at the Planck scale (Loll, 2019). Furthermore, a deeper connection between fractal topology and other areas of

mathematics, such as number theory (via Diophantine approximation) and algebraic topology, could unlock new theoretical structures (Khalili *et al.*, 2024).

- **Data Science and Visualization:** In an era of “big data,” fractal dimensions could serve as a powerful tool for characterizing the intrinsic complexity of high-dimensional datasets. This can aid in dimensionality reduction and the visualization of complex information structures, helping to reveal hidden patterns that would otherwise remain obscured (Andronache *et al.*, 2024).

6. Conclusion

Fractal topology has evolved from a mathematical curiosity into a fundamental language for describing and analyzing the irregular complexity that pervades the universe. Its core concepts—the fractional dimension, the intricate dance of connectedness and compactness, and the principle of self-similarity—provide a robust framework that transcends disciplinary boundaries. The field’s vitality is evident not only in its wide-ranging applications, from medicine to materials science, but also in the depth and difficulty of its open problems. Conjectures like MLC and the challenge of building a full theory of calculus on fractals represent grand challenges that continue to inspire mathematical innovation.

Looking forward, the synthesis of fractal topology with artificial intelligence and its application to the design of novel technologies ensures its continued relevance. As our tools for observation and computation grow more powerful, we are likely to discover that fractal patterns are even more ubiquitous than we currently imagine. The study of fractal topology is, therefore, more than an exploration of abstract shapes; it is a critical component in the ongoing scientific endeavor to understand the complex, structured, and beautifully intricate reality we inhabit.

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