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Mean Binary L-Cordial labeling

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ABSTRACT
Here, we have given the definition of Mean Binary L-Cordial labeling. We have shown that simple graphs like Path, Cycle and some subdivision of simple graphs admits Mean Binary L-Cordial labeling.

Keywords: Mean Binary L-Cordial, Subdivision of graphs, Actinia graph, Tadpole graph.

INTRODUCTION

L-Cordial labeling was first presented in [1]. In [2,3] they discussed about the L-Cordial labeling of some standard graphs. A detailed review of graph labeling is given in [5]. The definition of Binary L-Cordial labeling (B_{LCL}) is introduced in [6,7]. In this work we have defined the Mean Binary L-Cordial labeling and studied that some simple graphs satisfies Mean Binary L-Cordial labeling.

PRELIMINARIES

Definition 1.1 An edge subdivision is the insertion of a new vertex vj in the middle of an existing edge e=vivk accompanied by the joining of the original edge end points with the new vertex to form new edges $e^{'} = v_i v_j$ and $e^{"} = v_j v_k$

Definition 1.2 The (m,n) tadpole graph is a special type of graph consisting of a cycle graph on m (atleast 3) vertices and a path on n vertices, connected with a bridge.

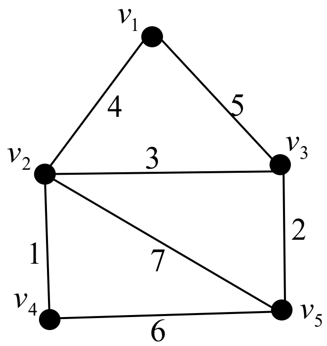
Definition 1.3 An actinia graph is a unicyclic graph formed by attaching a specific number of pendant edges to each vertex of a cycle.

We have given the new type of labeling.

Definition 2.1

If a graph G(V,E) has a bijection $h: E(G) \rightarrow \{1,2,\ldots |E|\}$ and for each vertex u, if $\frac{h(uv_i)+h(uv_j)}{2}$ has an odd integral part then assign the label as 1 and 0 for an even integral part where uv_i, uv_j have the smallest and the greatest h values. This graph is said to be Mean B_{LCL} if it also satisfies the condition that $|V_h(0)-V_h(1)| \leq 1$, where $V_h(0)$ indicate the number of u's with label 0 and $V_h(1)$, the number of u's with label 1. The graph is called Mean Binary L-cordial graph on the off chance that concedes Mean B_{LCL} .

Example 2.1.



$V_h(0,1) = (3,2)$

MAIN RESULTS

Theorem 3.1: Path graph admits Mean B_{LCL} .

Proof: Let $\{u_i\}_{i=1}^{n}$ denote the vertices of the path graph and let $\{e_i\}_{i=1}^{n-1}$ denote the edges of the path.

We define the bijection $h: E(G) \rightarrow \{1,2,..|E|\}$ as follows.

For $n \equiv 0,2(\bmod 4)$,

$h(e_1) = 2$

$h(e_2) = 1$

$h(e_i) = i; i = 3,4,..n-1$

We get, $|V_h(0)| = |V_h(1)| = \dfrac{n}{2}$

For $n \equiv 1,3(\bmod 4)$

$h(e_i) = i\,; i = 1,2,..n-1$

We get, $$|V_h(0)| = \frac{n-1}{2}$$ $$|V_h(1)| = \frac{n+1}{2}$$

Hence, the above labeling function satisfies the condition of Mean B_{LCL} .

Theorem 3.2: C_n satisfies Mean B_{LCL}

Proof: Let $\{u_i\}_{i=1}^{n}$ denote the vertices and $\{e_i\}_{i=1}^{n}$ denote the edges.

Define a bijection $h: E(G) \rightarrow \{1,2,..|E|\}$.

Label the edges of C_n as follows.

Case(1): $n \equiv 0,1,3(\bmod 4)$

$h(e_i) = i\,; i = 1,2,..n$

Case (2): $n \equiv 2(\bmod 4)$

$h(e_i) = i; i = 2,3,..n-1$

$h(e_1) = n$

$h(e_n) = 1$

By, labeling the vertices of the graph for both cases using the Mean B_{LCL} condition, we get $|V_h(0) - V_h(1)| \leq 1$.

Theorem 3.3: Subdivision of Star Graphs admit Mean B_{LCL} .

Proof: Let the set $\{v, u_1, u_2, ..u_n, u_1^{'}, u_2^{'}, ..u_n^{'}\}$ be the vertices of $SD(K_1, n)$ where $v, u_1, u_2, ..u_n$ are the vertices of $K_{1,n}$ *and* $\{u_1^{'}, u_2^{'}, ..u_n^{'}\}$ are the subdivision vertices of $K_{1,n}$.

Define a bijection $h: E(G) \to \{1,2,..|E|\}$

$h(vu_i^{'}) = 2i-1; i = 1,2,..n$

$h(u_i^{'}u_i) = 2i; i = 1,2,..n$

Label the vertices of the graph using Mean B_{LCL} definition we get, $|V_h(0) - V_h(1)| \le 1$

Theorem 3.4: Subdivision of actinia graph satisfies Mean B_{LCL} .

Proof: Let $\{u_1, u_2,..u_n\}$ *and* $\{v_1, v_2,..v_n\}$ denote the vertices of the actinia graph and Let $\{u_1^{'}, u_2^{'},..., u_n^{'}\}$ *and* $\{v_1^{'}, v_2^{'},...v_n^{'}\}$ be the subdivision vertices of the actinia graph.

Define a bijection $h: E(G) \to \{1,2,..|E|\}$.

Case1: When 'n' is even.

$h(u_i u_i^{'}) = 2[i(n)] + 2i - 1 \quad ; i = 1,2,..n$

$h(u_i^{'}u_{i+1}) = 2[i(n)] + 2i \quad ; i = 1,2,..n-1 \quad ; \quad h(u_n^{'}u_1) = 2(n^2) + 2n$

$h(u_1 v_i^{'}) = 2i - 1 \quad ; i = 1,2,..n$

$h(u_2 v_{n+i}^{'}) = (2n+2) + (2i-1) \quad ; i = 1,2,..n$

$h(u_3 v_{2n+i}^{'}) = (4n+4) + (2i-1) \quad ; i = 1,2,..n$

$h(u_4 v_{3n+i}^{'}) = (6n+6) + (2i-1) \quad ; i = 1,2,..n$

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$h(v_i v_i^{'}) = 2i \quad ; i = 1,2,..n$

$h(v_{n+i} v_{n+i}^{'}) = (2n+2) + 2i \quad ; i = 1,2,..n$

$h(v_{2n+i} v_{2n+i}^{'}) = (4n+4) + 2i \quad ; i = 1,2,..n$

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Case 2: When 'n' is odd.

$h(u_i u_i^{'}) = 2i - 1; i = 1,2,..n$

$h(u_i^{'}u_{i+1}) = 2i; i = 1,2,..n-1; \quad h(u_n u_1) = 2n$

$h(u_2 v_{n+i}^{'}) = 2n + (2i-1); i = 1,2,..n$

$h(u_3 v_{2n+i}^{'}) = 4n + (2i-1); i = 1,2,..n$

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$h(u_1 v_i^{'}) = 2n^2 + (2i-1); i = 1,2,..n$

$h(v_{n+i}^{'} v_{n+i}) = 2n + 2i; i = 1,2,..n$

$h(v_{2n+i}^{'} v_{2n+i}) = 4n + 2i; i = 1,2,..n$

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$h(v_i^{'} v_i) = 2n^2 + 2i \ ; \ i = 1,2,..n$

Now label the vertices of the graph using Mean B_{LCL} definition for both cases, we get $|V_h(0)| = |V_h(1)|$.
Illustration of Subdivision of A(3,3) is given in figure 1.

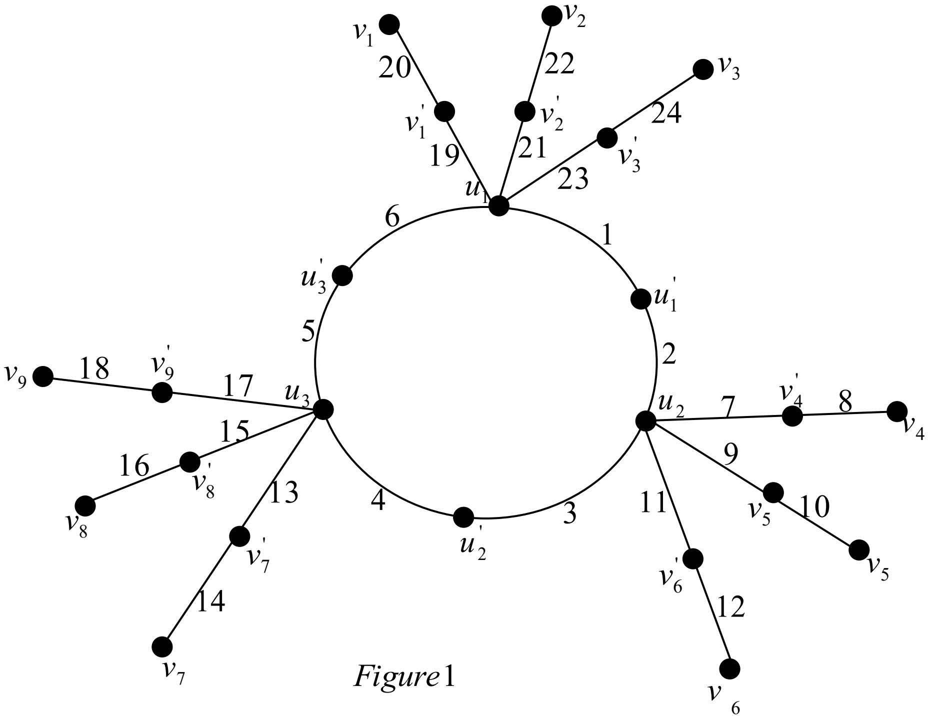

Figure 1

Theorem 3.5: Subdivision of Crown graph concedes Mean B_{LCL} for odd values of 'n'.

Proof: Let $\{u_i, v_i / i = 1,2,..n\}$ denote the set of vertices of the crown graph and $\{u_1', u_2', ..u_n', v_1', v_2', ...v_n'\}$ denote the set of subdivision vertices of the crown graph.

Define a bijection $h : E(G) \to \{1,2,..|E|\}$

$$h(u_i u_i') = 2i - 1 \quad ; i = 1,2,..n$$

$$h(u_i' u_{i+1}) = 2i \quad ; i = 1,2,..n-1 \ ; \ h(u_n u_1) = 2n$$

$$h(u_{i+1} v_{i+1}') = 2n + (2i-1) ; i = 1,2,...n-1$$

$$h(u_1 v_1') = 4n - 1 \quad ; \ h(v_1' v_1) = 4n$$

$$h(v_{i+1}' v_{i+1}) = 2n + 2i \quad ; i = 1,2,..n-1$$

Label the vertices of the graph using Mean B_{LCL} definition, we get $|V_h(0)| = |V_h(1)|$

Illustration of Subdivision of Crown(C3) is given in figure 2.

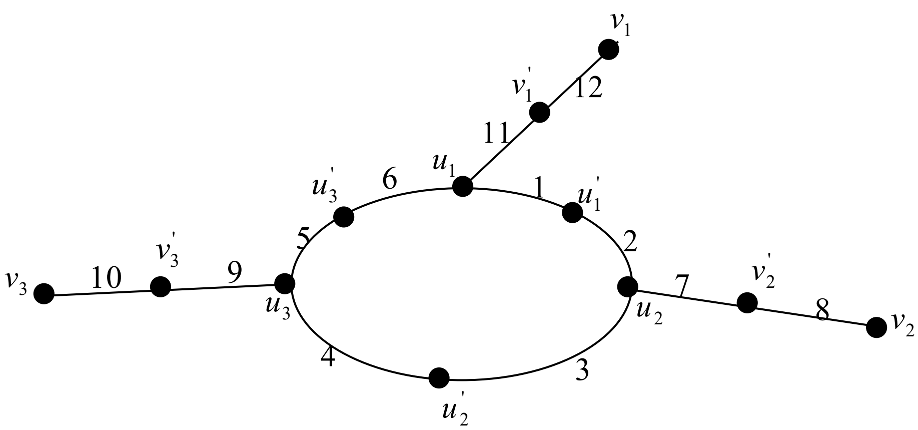

Figure 2

Theorem 3.6 Subdivision of Tadpole graph $(T_{n,n})$ satisfies Mean B_{LCL} .

Proof: Let $\{u_1,u_2,..u_n,u_1',u_2',..u_n'\}$ denote the set of vertices of the cycle in a tadpole graph and let $\{v_1,v_2,...v_n,v_1',v_2',....v_n'\}$ denote the set of vertices of the path in a tadpole graph where $u_1,u_2,..u_n,v_1,v_2,...v_n$ are the vertices of the tadpole graph and $u_1',u_2',..u_n',v_1',v_2',...v_n'$ are the subdivision vertices of the tadpole graph.

Define a bijection $h:E(G)\to\{1,2,..|E|\}$

Case 1: When 'n' is even

$h(u_1v_1') \quad =1$

$h(v_i'v_i) \quad =2i \quad ;i=1,2,...n$

$h(v_iv_{i+1}') \ =2i+1 \ ;i=1,2,..n-1$

$h(u_iu_i') \quad =2n+(2i-1);i=1,2,..n$

$h(u_i'u_{i+1}) \ =2n+2i \quad ;i=1,2,..n-1 \ ; \ h(u_n'u_1)=4n$

Case 2: When 'n' is odd

$h(u_iu_i') \quad =2i-1 \ ;i=1,2,..n$

$h(u_i'u_{i+1}) \ =2i \quad ;i=1,2,..n-1 \ ; \ h(u_n'u_1)=2n$

$h(v_i'v_i) \quad =2n+2i;i=1,2,..n$

$h(u_1v_1') \quad =2n+1$

$h(v_iv_{i+1}') \ =2n+(2i+1);i=1,2,..n-1$

Label the vertices of the graph for both cases using Mean B_{LCL} definition, we get $|V_h(0)|=|V_h(1)|$.

Illustration of Subdivision of (T4,4) is given in figure 3.

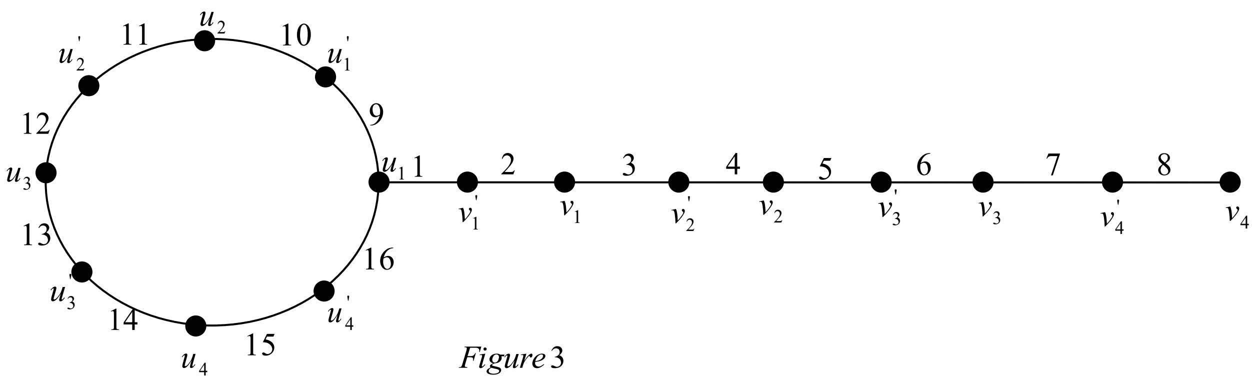

Figure 3

Theorem 3.7 Subdivision of Comb graph of odd values of n satisfies Mean B_{LCL} .

Proof: Let $\{u_i,v_i / i=1,2,...n\}$ denote the set of vertices of the comb graph and let $\{u_1',u_2',..u_n',v_1',v_2',..v_n'\}$ denote the subdivision vertices of the comb graph.

Define a bijection $h:E(G)\to\{1,2,..|E|\}$.

$h(u_iu_i') \quad =2i-1 \quad ;i=1,2,..n-1$

$h(u_i'u_{i+1}) \ =2i \quad ;i=1,2,..n-1$

$h(u_iv_i') \quad =(2n-3)+2i \ ;i=1,2,...n$

$h(v_i'v_i) \quad =(2n-2)+2i \ ;i=1,2,..n$

Now label the vertices of the graph using Mean B_{LCL} definition we get, $|V_h(1)|=|V_h(0)|+1$.

Illustration of Subdivision of $P_3\circ K_1$ is given in figure 4.

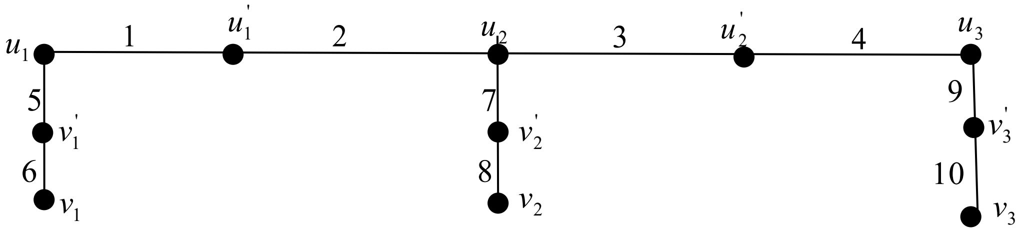

*Figure*4

Theorem 3.8 Subdivision of Dumbell graph concedes Mean B_{LCL}.

Proof: Let $\{u_1,u_2,..u_n,u_1',u_2',..u_n'\}$ denote the set of vertices of cycle C_1 in a dumbell graph. Let $\{v_1,v_2,...v_n,v_1',v_2',...v_n'\}$ denote the set of vertices of cycle C_2 where $\{u_1,u_2,..u_n,v_1,v_2,...v_n\}$ are the set of vertices of the dumbbell graph and $\{u_1',u_2',..u_n',v_1',v_2',...v_n'\}$ denote the set of subdivision vertices of the dumbbell graph. Let w be the subdivision vertex of the edges u_1v_1.

Define a bijection $h: E(G) \to \{1,2,..|E|\}$.

$h(u_iu_i') = 2i-1 \;; i=1,2,...n$

$h(u_i'u_{i+1}) = 2i \;; i=1,2,..n-1 \;; h(u_n'u_1) = 2n$

$h(u_1w) = 2n+1 \;;\; h(wv_1) = 2(2n+1)$

$h(v_iv_i') = 2n+2i \;; i=1,2,..n$

$h(v_i'v_{i+1}) = 2n+(2i+1) \;; i=1,2,..n-1 \;;\; h(v_n'v_1) = 4n+1$

By labeling the vertices of the graph using Mean B_{LCL} definition we get, $|V_h(0)-V_h(1)| \le 1$.

Conclusion

In this paper we have defined Mean B_{LCL} and have shown some simple graphs admit Mean B_{LCL}.

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