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Hesitant Intuitionistic Fuzzy Strongly Prime Submodule

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Abstract. In this paper, we investigate the concept of hesitant intuitionistic ƒ-strongly prime submodules of an module M. The fundamental properties of this concept are systematically investigated. In particular, it is proved that the intersection of any two hesitant intuitionistic ƒ-strongly prime submodules is again a hesitant intuitionistic ƒ-strongly prime submodule. Furthermore, it is shown that every hesitant intuitionistic ƒ-strongly prime submodule is necessarily a hesitant intuitionistic ƒ-prime submodule. A characterization of hesitant intuitionistic ƒ-strongly prime submodules is also obtained, and their connection with classical strongly prime submodules is examined through the characteristic hesitant intuitionistic ƒ-set. The obtained results extend existing theories on ƒ -intuitionistic ƒ-submodules and provide a broader theoretical framework for studying algebraic structures under hesitant intuitionistic ƒ-environments.

Keywords: Hesitant intuitionistic ƒ - set, hesitant ƒ - submodule, hesitant intuitionistic ƒ - submodule, hesitant intuitionistic ƒ - strongly prime submodule.

1.iIntroduction

The theory of ƒ -sets, introduced by Zadeh [12], has become one of the most influential mathematical frameworks for modeling uncertainty and vagueness. To overcome the limitations of classical ƒ - sets, Atanassov [2] proposed intuitionistic ƒ - sets. Later, Torra [11] introduced hesitant fuzzy sets to represent hesitation in membership evaluation. The integration of these concepts has led to hesitant intuitionistic ƒ - sets, which provide a more flexible framework for handling complex uncertain information. In module theory, prime and strongly prime submodules have been extensively investigated. Their ƒ - extensions were studied by Bharbet and Kumar [3], while several properties of intuitionistic ƒ - prime submodules were established by Poonam K. S. and Gagandeep K. [7]. Moreover, Azizi [1] and Khashan H. A. [5] contributed to the development of prime and almost prime submodules. More recently, Fadhil M. J. Mohammed and R. Hadi [4] introduced hesitant ƒ - strongly prime submodules. Despite these developments, several structural properties and characterizations of hesitant intuitionistic ƒ - strongly prime submodules remain unexplored. Motivated by this gap, the present paper introduces the notion of hesitant intuitionistic ƒ - strongly prime submodules, establishes their fundamental properties, and provides new characterizations that extend and generalize existing results in the literature.

2.iPreliminaries

In this section. We list some definitions, and upshots needed in the advance sections.

Definition(i2.1) [12] : Let Ӿ be a reference set, and let $\mathbb{I} = [0,1]$. A **ƒ-set** Ɣ is a function $Ɣ : Ӿ \to \mathbb{I}$. The value $Ɣ(y)$ signifies the degree of membership of y to Ӿ. All **ƒ-sets** are collectively denoted by $\mathbb{I}^{y}$.

Definition(2.2)[2]: An **intuitionistic ƒ-set** E of reference set Ӿ is defined as :
$E = \{< y, Ɣ_E(y), ƣ_E(y)>: y \in Ӿ\}$. Where $Ɣ_E: Ӿ \to [0,1]$iand $ƣ_E: Ӿ \to [0,1]$. Signifies the degrees of membership and non-membership of all y ∈ Ӿ in the set E, respectively. These functions must satisfy the condition : $0 \le Ɣ_E(y) + ƣ_E(y) \le 1$ for y ∈ Ӿ. The gathering of all intuitionistic ƒ-sets on Ӿ is denoted by IF(Ӿ). Also: $\pi_E(y) = 1 - Ɣ_E(y) - ƣ_E(y)$, ∀y ∈ Ӿ the intuitionistic index or hesitancy degree of E. It is obvious that, $0 \le \pi_E \le 1$, for y∈ Ӿ .

Definition(2.3)[11]: A **hesitant ʄ-set** E of reference set Ӿ is a map $h: Ӿ \to P([0,1])$. Where $E = \{< y, h_E(y) >: iy \in Ӿ\}$. And $h_E(y)$ is a set of some values in [0,1]. Such that when applied to elements of Ӿ, it returns a subset of the interval [0,1].

For any **hesitant intuitionistic ʄ -set** A and for every $y \in Ӿ$, let $Ɣ̀_{h_A}(y)$ denote the set of all possible membership degrees of y, and $ɋ̀_{h_A}(y)$ denote the set of all possible non-membership degrees of y
Furthermore,

$$Ɣ̀_{h_A}{}^{-}(y) = \min Ɣ̀_{h_A}(y), \qquad Ɣ̀_{h_A}{}^{+}(y) = \max Ɣ̀_{h_A}(y).$$

$$ɋ̀_{h_A}{}^{-}(y) = \min ɋ̀_{h_A}(y), \qquad ɋ̀_{h_A}{}^{+}(y) = \max ɋ̀_{h_A}(y).$$

Definition(2.4)[10]: Let Ɣ and ɋ are two maps from a reference set Ӿ to the association of subsets of 𝕀. And let A= {<y, $Ɣ̀_{h_A}(y)$, $ɋ̀_{h_A}(y)$ > / y ∈ Ӿ }, then A is called a hesitant intuitionistic ʄ -set (ḤỊḞs) on Ӿ, where values $Ɣ̀_{h_A}(y)$ and $ɋ̀_{h_A}(y)$ represent the potential membership and non-membership degrees of y∈ Ӿ to the set A respectively. Which satisfy $Ɣ̀_{h_A}{}^{-}(y) + ɋ̀_{h_A}{}^{+}(y) \le 1$ and $Ɣ̀_{h_A}{}^{+}(y) + ɋ̀_{h_A}{}^{-}(y) \le 1$.

Definition (2.5)[6]: Let A and B are two be hesitant intuitionistic ʄ -sets of the forms A= $\{< y, Ɣ̀_{h_A}(y), ɋ̀_{h_A}(y) >/ y \in Ӿ\}$. B = $\{< y, Ɣ̀_{h_B}(y), ɋ̀_{h_B}(y)i >/ y \in Ӿ\}$ on Ӿ then:

a) iA ⊂ B if and only if $Ɣ̀_{h_A}(y) \subseteq Ɣ̀_{h_B}(y)$and $ɋ̀_{h_A}(y) \supseteq ɋ̀_{h_B}(y)$ for y∈ Ӿ.

b) A = B if and only if A ⊆ B and iB ⊇ A.

c) $A^c = \{< y, ɋ̀_{h_A}(y), Ɣ̀_{h_A}(y) >: y \in Ӿ\}$.

d) $A \cup B = \{< y, Ɣ̀_{h_A}(y) \cup Ɣ̀_{h_B}(y), ɋ̀_{h_A}(y) \cap ɋ̀_{h_B}(y) >/ y \in Ӿ\}$

Where:

$= \cup_{y_1 \in Ɣ̀_{h_A}(y), y_2 \in Ɣ̀_{h_B}(y)} \max\{y_1, y_2\}$ and $(Ɣ̀_{h_A} \cup Ɣ̀_{h_B})(y)$

$= \cup_{y_1 \in ɋ̀_{h_A}(y), y_2 \in ɋ̀_{h_B}(y)} \min\{y_1, y_2\}.(ɋ̀_{h_A} \cap ɋ_{h_B})(y)$

e) $A \cap B = \{< y, Ɣ̀_{h_A}(y) \cap Ɣ̀_{h_B}(y), ɋ̀_{h_A}(y) \cup ɋ̀_{h_B}(y) >/ y \in Ӿ\}$.

Where:

$= \cup_{y_1 \in Ɣ̀_{h_A}(y), y_2 \in Ɣ̀_{h_B}(y)} \min\{y_1, y_2\}$ and $(Ɣ̀_{h_A} \cap Ɣ̀_{h_B})(y)$

$(ɋ̀_{h_A} \cup ɋ̀_{h_B})(y) = \cup_{y_1 \in ɋ̀_{h_A}(y), y_2 \in ɋ̀_{h_B}(y)} \max\{y_1, y_2\}$.

Definition (2.6)[8]: Let $x_{(\alpha,\beta)}$ be a hesitant intuitionistic ʄ -point (ḤỊḞP) of Ӿ such that αi, β ⊆ [0, 1], defined by : $x_{(\alpha,\beta)}(y) = \begin{cases} i(\alpha, \beta) & : \quad iy = x \\ i(\emptyset, [0,1]) & : \quad iy \neq x \end{cases}$

In this case x is said to be the support of $x_{(\alpha,\beta)}$, and α,β are said membership value and non- membership value of $x_{(\alpha,\beta)}$ respectively.

A ḤỊḞP $x_{(\alpha,\beta)}$ is called belong to a ḤỊḞs Aiof Ӿ meant by $x_{(\alpha,\beta)} \in A$, $\alpha \subseteq Ɣ̀_{h_A}(x)$iand $\beta \supseteq ɋ̀_{h_A}(x)$.

Definition(2.7)[8]: A hesitant intuitionistic ʄ -set A in Ӿ, such that (α, β)-level set defined by : $A_{H_{(\alpha,\beta)}} = \{y \in Ӿ: Ɣ̀_{h_A}(y) \supseteq \alpha, ɋ̀_{h_A}(y) \subseteq \beta\}$. Where α, β ⊆ [0,1].

Definition(2.8)[8]: Let A subset of Ӿ. The characteristic hesitant intuitionistic ʄ -set of A, denoted by χ_{h_A}, is defined as: $\chi_{h_A}(y) = < Ɣ_{h\chi_A}(y), ɋ_{h\chi_A}(y): y \in Ӿ >$, where:

$$Ɣ_{h\chi_A}(y) = \begin{cases} [0,1] & \text{if } y \in A \\ \emptyset & \text{if } y \notin A \end{cases} \qquad ɋ_{h\chi_A}(y) = \begin{cases} \emptyset & \text{if } y \in A \\ [0,1] & \text{if } y \notin A \end{cases}$$

Definition(2.9)[4] : A hesitant ʄ -set h of M is called hesitant ʄ -module (ḤḞṂ) of R-module M if for a, b ∈ M, s ∈ R.

(i) $h(a - b) \supseteq h(a) \cap h(b)$ (ii) $h(sa) \supseteq h(a)$

Definition (2.10)[3] : A sub-module N of M is called strongly prime submodule (in short, S-ꝑ-ṢṂ) of Ṃ if rx ∈ N for r ∈ R, r ≠ 0 and x ∈ M , then x ∈ N.

Definition(2.11)[9]: A hesitant intuitionistic ʄ- set A = ($Ɣ̀_{h_A}$, $ɋ̀_{h_A}$) of M is called hesitant intuitionistic ʄ-module (HIFM) if:

1) $Ɣ̀_{h_A}(\theta) = [0,1]$, $ɋ̀_{h_A}(\theta) = \emptyset$, where θ is a zero element of M,

2) $Ɣ̀_{h_A}(x + y) \supseteq Ɣ̀_{h_A}(x) \cap Ɣ̀_{h_A}(y)$, and $ɋ̀_{h_A}(x + y) \subseteq ɋ̀_{h_A}(x) \cup ɋ̀_{ℏ_E}(y)$,∀x, y ∈M.

3) $Ɣ̀_{h_A}(sx) \supseteq Ɣ̀_{h_A}(x)$ and $ɋ̀_{h_A}(sx) \subseteq ɋ̀_{h_A}(x)$, ∀ x∈M, s ∈ R.

Definition(2.12)[9]: A hesitant intuitionistic **ʄ**-submodule A of Ṃ is said to be a hesitant intuitionistic **ʄ** -prime submodule (ꝑ-ḤỊḞṢṂ(Ṃ)) of Ṃ, if for each $r_{(\delta,\gamma)} \in$ ḤỊḞP(R), $x_{(\alpha,\beta)} \in$ ḤỊḞP(M) {r ∈ R, x ∈ Ṃ . α, β, δ, γ ⊆ [0, 1]} Such that $r_{(\delta,\gamma)} x_{(\alpha,\beta)} \subseteq A$, then either $x_{(\alpha,\beta)} \subseteq A$ or $r_{(\delta,\gamma)} \chi_{h_M} \subseteq A$.

Definition(2.13)[9] : Let $r_{(\delta,\gamma)}$ be a hesitant intuitionistic **ʄ**- point of R and $y_{(\sigma,\tau)}$, $x_{(\alpha,\beta)}$ be hesitant intuitionistic **ʄ** -points of Ṃ, then:

(1) $r_{(\delta,\gamma)}x_{(\alpha,\beta)} = (rx)_{(\delta\cap\alpha,\gamma\cup\beta)}$. (2) $x_{(\alpha,\beta)} + y_{(\sigma,\tau)} = (x+y)_{(\alpha\cap\sigma,\beta\cup\tau)}$

3.Hesitant intuitionistic fuzzy strongly prime submodule

In this section we define a hesitant intuitionistic ʄ -strongly prime submodule of M and the relationships that include this concept with some of their properties and results.

Definition(3.1): A hesitant intuitionistic ʄ -submodule A is called a hesitant intuitionistic ʄ -strongly prime submodule (S-ꝑ-ḤḬḞṢṂ(M)), if for each $r_{(\delta,\gamma)} \in$ ḤḬḞ$P(R)$, $x_{(\alpha,\beta)} \in$ ḤḬḞ$P(M)$ $[r \in R, x \in M. \alpha,\beta,\delta,\gamma \subseteq [0,1]]$ *such that* $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq A$ implies that $x_{(\alpha,\beta)} \subseteq A$.

Theorem (3.2): Let A and B be S- ꝑ-ḤḬḞṢṂ(M). Then A ∩ B be a S- ꝑ-ḤḬḞṢṂ(M) .

Proof: Suppose that A and B are S- ꝑ-ḤḬḞṢṂ(M), and for each $r_{(\delta,\gamma)} \in$ ḤḬḞ$P(R)$, $x_{(\alpha,\beta)} \in$ ḤḬḞ$P(M)$ $[r \in R, x \in M . \alpha,\beta,\delta,\gamma \subseteq [0,1]]$ *such that* $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq A \cap$ B.

Then $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq A$ *and* $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq$ B.

Since A and B be S- ꝑ-ḤḬḞṢṂ(M).

Thus $x_{(\alpha,\beta)} \subseteq$ A and $x_{(\alpha,\beta)} \subseteq$ B. So, $x_{(\alpha,\beta)} \subseteq$ A ∩ B.

Implies A ∩ B be a S- ꝑ-ḤḬḞṢṂ(M).

Corollary (3.3): Let $\{A_{h_i}, i \in I\}$ be a family of a S-ꝑ-ḤḬḞṢṂ(M), then $(\bigcap_{i\in I} A_{h_i})$ is a S-ꝑ-ḤḬḞṢṂ(M).

Theorem (3.4) : Let A and B be S-ꝑ-ḤḬḞṢṂ(M). Then A ∩ B is a ꝑ-ḤḬḞṢṂ(M).

Proof : Suppose that A and B are S- ꝑ-ḤḬḞṢṂ(M), and for each $r_{(\delta,\gamma)} \in$ ḤḬḞ$P(R)$, $x_{(\alpha,\beta)} \in$ ḤḬḞ$P(M)$ $[r \in R, x \in$ Ṃ $. \alpha,\beta,\delta,\gamma \subseteq [0,1]]$ *such that* $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq A \cap$ B.

Then $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq A$ *and* $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq$ B.

Since A and B be S- ꝑ-ḤḬḞṢṂ(M).

Thus $x_{(\alpha,\beta)} \subseteq$ A and $x_{(\alpha,\beta)} \subseteq$ B. So, $x_{(\alpha,\beta)} \subseteq$ A ∩ B.

So, A ∩ B be a ꝑ-ḤḬḞṢṂ(M).

Definition (3.5): The support of hesitant intuitionistic ʄ -set A on Ӿ, denoted by A^*, is defined as the set:

$A^* = \{a \in$ Ӿ $: \emptyset \subseteq \grave{Ɣ}_{h_A}(a), ꝗ\grave{}_{h_A}(a) \subseteq [0,1]\}$.

Theorem (3.6) : let A be a S-ꝑ-ḤḬḞṢṂ(M), then A^* is a S-ꝑ-ṢṂ(M).

Proof : Let r ∈ R and $m \in M$ such that $rm \in A^*$ then,

$\subseteq \grave{Ɣ}_{h_A}(rm)$ and $ꝗ\grave{}_{h_A}(rm) \subseteq [0,1].\emptyset$

We assume that $\grave{Ɣ}_{h_A}(rm) = \alpha$ *and* $ꝗ\grave{}_{h_A}(rm) = \beta$, where $\alpha,\beta \subseteq [0,1]$. So, $(rm)_{(\alpha,\beta)} \subseteq A$.

Then $(r)_{(\alpha,\beta)}$ $(m)_{(\alpha,\beta)} \subseteq$ A [by def. 2.13] .

Since A is a S-ꝑ-ḤḬḞṢṂ(M).

Then m $_{(\alpha,\beta)} \subseteq$ A implies $\alpha \subseteq \grave{Ɣ}_{h_A}(m)$ and $ꝗ\grave{}_{h_A}(m) \subseteq \beta$ [by def. 2.6].

Since $\emptyset \subseteq \alpha$ and $\beta \subseteq [0,1]$. Implies $\emptyset \subseteq \alpha \subseteq \grave{Ɣ}_{h_A}(m)$ and $ꝗ\grave{}_{h_A}(m) \subseteq \beta \subseteq [0,1]$.

So, $\emptyset \subseteq \grave{Ɣ}_{h_A}(m)$ and $ꝗ\grave{}_{h_A}(m) \subseteq [0,1]$. Therefore, $m \in A^*$.

Then A^* is a S-ꝑ-ṢṂ(M).

Theorem (3.7) : Let A be a S-ꝑ-ḤḬḞṢṂ(Ṃ), if $\grave{Ɣ}_{h_A}(0) \supseteq \alpha$, $ꝗ\grave{}_{h_A}(0) \subseteq \beta$ then $A_{H_{(\alpha,\beta)}}$ is S-ꝑ-ṢṂ(M).

Proof : Let r ∈ R, x ∈ M such that rx ∈ $A_{H_{(\alpha,\beta)}}$.

Since rx ∈ $A_{H_{(\alpha,\beta)}}$ implies $\grave{Ɣ}_{h_A}(rx) \supseteq \alpha$ and $ꝗ\grave{}_{h_A}(rx) \subseteq \beta$ [by def. 2.7].

So, $(rx)_{(\alpha,\beta)} \subseteq$ A implies $r_{(\alpha,\beta)}x_{(\alpha,\beta)} \subseteq$ A [by def. 2.13].

Since A be a S-ꝑ-ḤḬḞṢṂ(M).

Thus, $x_{(\alpha,\beta)} \subseteq$ A .

Implies $\grave{Ɣ}_{h_A}(x) \supseteq \alpha$ and $ꝗ\grave{}_{h_A}(x) \subseteq \beta$.

Then x ∈ $A_{H_{(\alpha,\beta)}}$.

Therefore, $A_{H_{(\alpha,\beta)}}$ is a S-ꝑ-ṢṂ(M) .

Proposition (3.8) : Every S-ꝑ-ḤḬḞṢṂ(M) is a ꝑ-ḤḬḞṢṂ(M).

Proof : Let A is a S-ꝑ-ḤḬḞṢṂ(M), and let $r_{(\delta,\gamma)} \in$ ḤḬḞP(R), $x_{(\alpha,\beta)} \in$ ḤḬḞP(M) $[r \in R, x \in M . \alpha,\beta,\delta,\gamma \subseteq [0,1]]$ such that $r_{(\delta,\gamma)}x_{(\alpha,\beta)} \subseteq$ A

Since A is a S-ꝑ-ḤḬḞṢṂ(M). Then $x_{(\alpha,\beta)} \subseteq$ A.

Therefore A isꝑ − ḤḬḞṢṂ(M) .

Remark (3.9) : The proposition above cannot be reversed in general. For example .

Let Ṃ = Z as a Z-module and let A = <$\grave{Ɣ}_{h_A}$, $ꝗ\grave{}_{h_A}$ >, such that :

$$\grave{Ɣ}_{h_A}(a) = \begin{cases} [0,1] & \text{if} \quad a \in N = 5Z \\ \emptyset & \text{if} \quad \text{otherwise} \end{cases} \qquad ꝗ\grave{}_{h_A}(a) = \begin{cases} \emptyset & \text{if} \quad a \in N = 5Z \\ [0,1] & \text{if} \quad \text{otherwise} \end{cases}$$

Since A is a ᵽ-ḤỊ̰ḞṢṂ(M). Then $5_{[0.1,0.4]}7_{[0.4,0.6]} = 35_{[0.1,0.4]} \subseteq A$, because
$\text{Ɣ}_{h_A}(35) = [0,1] \supseteq [0.1,0.4]$ and $\text{ꝗ}_{h_A}(35) = \emptyset \subseteq [0.1,0.4]$.
But $7_{[0.4,0.6]} \nsubseteq A$ because $\text{Ɣ}_{h_A}(7) = \emptyset \nsupseteq [0.4,0.6]$ and $\text{ꝗ}_{h_A}(7) = [0,1] \nsubseteq [0.4,0.6]$.
Therefore A is not S-ᵽ-ḤỊ̰ḞṢṂ(M).

4. Conclusion

In conclusion, we study the main definitions and theorems related to hesitant intuitionistic **ʄ** -strongly prime submodule and we have obtained new results, which is the relationship between hesitant intuitionistic **ʄ** -strongly prime submodule and hesitant intuitionistic **ʄ** - prime submodule such that every hesitant intuitionistic **ʄ** -strongly prime submodule is hesitant intuitionistic **ʄ** - prime submodule of R-module M.

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