

# Unified Lyapunov-Based Integral Nonsingular Fast Terminal Sliding Mode Control for Nonlinear Systems with Quantitative Performance Evaluation

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## Abstract

This paper presents a Unified Lyapunov-Based Integral Nonsingular Fast Terminal Sliding Mode Control (ULI-NFTSMC) framework for tracking nonlinear systems subject to parametric uncertainties and bounded matched disturbances. The proposed controller incorporates proportional and integral error dynamics into a nonsingular fast terminal sliding surface and employs a bounded integral-state mechanism to limit integral accumulation during large transients and actuator saturation. The formulation is developed using a matched-uncertainty representation of the nonlinear system, and a Lyapunov-based analysis establishes a sufficient switching-gain condition for finite-time reachability of the proposed sliding variable. The controller design combines integral error augmentation with nonsingular fast terminal dynamics while avoiding the singularity associated with conventional terminal sliding-mode formulations. The proposed approach is evaluated using a nonlinear coupled twin-tank liquid-level process under nominal operating conditions and under parametric uncertainty with an external disturbance. Tracking performance and control characteristics are assessed using integral absolute error (IAE), integral squared error (ISE), control smoothness index (CSI), control requirement coefficient (CRC), and total normalized control-effort index (TECI). The simulation study demonstrates the applicability of the proposed ULI-NFTSMC framework to nonlinear uncertain process-control systems and provides quantitative measures of tracking accuracy, control-signal variation, and control effort.

**Keyword:** Nonsingular fast terminal sliding mode control; Integral sliding mode control; Robust nonlinear control; Finite-time reachability; Parametric uncertainty; Matched disturbances; Lyapunov stability; Quantitative performance evaluation.

## 1. Introduction

This section presents the background and motivation of the study, highlights the novelty of the proposed work, summarizes the main contributions, and outlines the structure of the article.

### 1.1 Introduction

Robust tracking of nonlinear dynamical systems remains an important problem in control engineering because practical systems frequently exhibit nonlinear dynamics, parametric variations, actuator limitations, and external disturbances. These characteristics arise in applications including process control, electric drives, robotics, power electronics, and other industrial systems. In agricultural and related operational environments, AI-enabled systems can further support decision-making, resource optimization, and risk reduction, highlighting the need for reliable control and automation strategies in complex systems [1]. Artificial intelligence and machine learning can facilitate early animal disease detection, reduce the workload associated with conventional disease-monitoring practices, and raise important ethical considerations regarding their application in veterinary healthcare [2]. Proportional-integral-derivative (PID) controllers are widely used because of their simple structure and practical implementation; however, their performance can deteriorate when significant nonlinearities, parameter variations, or disturbances are present. These limitations have motivated the development of nonlinear robust control techniques.

Sliding mode control (SMC) [3,5] is a well-established robust-control methodology for uncertain nonlinear systems. By designing an appropriate sliding manifold and driving the system trajectory toward it, SMC can provide robustness against a class of matched uncertainties and disturbances. However, the discontinuous switching action used in conventional SMC can produce high-frequency chattering, which may excite

unmodeled dynamics and increase actuator activity. Consequently, considerable research has focused on developing sliding-mode formulations that improve convergence characteristics while addressing singularity and chattering-related limitations.

Terminal sliding mode control (TSMC) introduced nonlinear sliding manifolds to obtain finite-time convergence characteristics. Classical TSMC, however, can exhibit singularity problems associated with the nonlinear terms in the control law. Nonsingular terminal sliding mode (NTSM) formulations were subsequently developed to avoid this difficulty while retaining finite-time convergence properties [6,7]. Fast terminal and nonsingular fast terminal sliding-mode formulations further modify the sliding manifold to improve convergence characteristics [15,16,19,20]. These developments have resulted in a broad range of NFTSMC applications to nonlinear uncertain systems.

Process-control applications may also involve unsymmetrical and multivariable dynamics. Unsymmetrical-process control has been studied using systematic tuning approaches [28], while fast terminal sliding-mode control has been applied to multivariable process systems [30]. These studies further motivate robust nonlinear control formulations for practical process systems.

Integral augmentation has also been investigated in terminal sliding-mode control to improve tracking accuracy and disturbance rejection. Li et al. proposed a fast nonsingular integral terminal sliding-mode controller for nonlinear dynamical systems using a power-integral sliding surface [21]. Qiao and Zhang subsequently developed an adaptive nonsingular integral terminal sliding-mode tracking controller for systems with dynamic uncertainties and time-varying disturbances [22]. These studies establish that integral augmentation can be combined with nonsingular terminal sliding-mode structures while retaining finite-time tracking properties. Accordingly, the contribution of the present work is not the introduction of integral action into nonsingular terminal sliding control itself, but the development and unified analysis of a proportional–integral augmented nonsingular fast terminal sliding-mode formulation incorporating a bounded integral-state mechanism, a Lyapunov-based finite-time reachability condition, and a quantitative evaluation framework for nonlinear process control.

Recent studies have further extended NFTSMC to a variety of nonlinear engineering applications. These include fault-tolerant UAV control, high-speed nanopositioning, islanded photovoltaic microgrids, and vehicle control under actuator faults [29–32]. More recent developments have addressed actuator constraints in underwater gliders, adaptive yaw-moment control of electric vehicles, disturbance-observer-based control of permanent-magnet synchronous motors, and learning-assisted trajectory tracking of robotic manipulators [35–38]. These developments demonstrate the continuing applicability of NFTSMC to uncertain nonlinear systems and highlight ongoing efforts to improve robustness, convergence characteristics, disturbance rejection, actuator handling, and control adaptability.

The proposed Unified Lyapunov-Based Integral Nonsingular Fast Terminal Sliding Mode Control (ULI-NFTSMC) incorporates proportional and integral error dynamics directly into the sliding surface. The integral state is bounded to limit excessive accumulation during large transients and actuator saturation. The resulting controller is formulated using a matched-uncertainty representation of the nonlinear system, allowing a sufficient switching-gain condition to be derived through Lyapunov analysis. The analysis specifically establishes finite-time reachability of the proposed sliding variable under the stated uncertainty and control-gain assumptions.

To evaluate the proposed formulation, a coupled twin-tank liquid-level process is considered as a nonlinear benchmark. Coupled-tank systems have been widely investigated for nonlinear and robust control studies because their liquid-level dynamics contain nonlinear flow relationships and coupled interactions [25–27]. In this work, the proposed controller is evaluated under nominal conditions and under parametric uncertainty with an external disturbance. In addition to conventional tracking measures, the study employs integral absolute error (IAE), integral squared error (ISE), control smoothness index (CSI), control requirement coefficient (CRC), and a total normalized control-effort index (TECI) to provide quantitative measures of tracking and control performance.

The main contributions of this study are therefore summarized as follows:

A proportional–integral augmented nonsingular fast terminal sliding-mode formulation is developed for nonlinear systems with bounded matched uncertainties and disturbances.

A bounded integral-state mechanism is incorporated into the controller formulation to limit integral accumulation during large transients and actuator saturation.

A Lyapunov-based analysis is developed to establish a sufficient switching-gain condition for finite-time reachability of the proposed sliding variable.

A quantitative evaluation framework based on tracking-error and control-effort indices is applied to a nonlinear coupled twin-tank liquid-level process under nominal and uncertainty-and-disturbance conditions.

The remainder of the paper is organized as follows. Section 2 formulates the nonlinear uncertain control problem and states the required assumptions. Section 3 presents the proposed ULI-NFTSMC formulation and control-law development. Section 4 establishes the Lyapunov-based finite-time reachability analysis. Section 5 provides parameter-selection guidelines. Section 6 presents the twin-tank simulation study and quantitative performance evaluation. Finally, Section 7 concludes the paper.

## 2. PROBLEM FORMULATION

Consider a general class of nonlinear perturbed systems represented in controllable canonical form [1,4,5]. The nonlinear uncertain system is described by

$$\dot{x}_i(t) = x_{i+1}(t), \quad i = 1, 2, \dots, n-1 \quad (1a)$$

and

$$\dot{x}_n(t) = f(x, t) + \Delta f(x, t) + [g(x, t) + \Delta g(x, t)]u(t) + d_{ext}(t) \quad (1b)$$

With

$$y(t) = x_1(t) \quad (1c)$$

where

$$x(t) = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$$

is the state vector,  $u(t)$  is the control input,  $f(x, t)$  represents the nominal nonlinear system dynamics, and  $g(x, t)$  denotes the nominal control gain. The terms  $\Delta f(x, t)$  and  $\Delta g(x, t)$  represent uncertainties in the system dynamics and control gain, respectively, while  $d_{ext}(t)$  represents an external disturbance acting on the system.

### 2.1 Uncertainty Representation

To facilitate the controller design, the uncertainties in the system dynamics and control gain are represented using multiplicative uncertainty terms [3,6,7,20].

Assumption 1

The uncertainties  $\Delta f(x, t)$  and  $\Delta g(x, t)$  satisfy,

$$\Delta f(x, t) = f(x, t)\bar{f} \quad (2a)$$

and

$$\Delta g(x, t) = g(x, t)\bar{g} \quad (2b)$$

where  $\bar{f}(x, t)$  and  $\bar{g}(x, t)$  are unknown but bounded uncertainty functions satisfying

$$|\bar{f}| \leq \delta_f \quad (2c)$$

and

$$|\bar{g}| \leq \delta_g < 1, \quad (2d)$$

where  $\delta_f$  and  $\delta_g$  are unknown finite positive constants.

The condition  $\delta_g < 1$  ensures that the uncertain control gain does not change sign with respect to the nominal control gain, since

$$g(x, t) + \Delta g(x, t) = g(x, t)[1 + \bar{g}(x, t)] \quad (2e)$$

and therefore

$$1 + \bar{g}(x, t) > 0. \quad (2f)$$

This assumption is required to maintain a well-defined control channel during controller design.

### 2.2 Nonzero Control Gain

For the proposed sliding-mode controller, the control gain must remain nonzero over the operating region.

Assumption 2

There exist positive finite constants  $g_{\min}$ ,  $g_{\max}$ , and  $f_{\max}$  such that

$$g_{\min} \leq |g(x, t)| \leq g_{\max} \quad (3)$$

and

$$|f(x, t)| \leq f_{\max} \quad (4)$$

where  $g_{\min} > 0$ ,  $g_{\max} > 0$ , and  $f_{\max} > 0$  are finite constants.

The lower bound  $g_{\min} > 0$  ensures that the control gain remains nonzero and that  $g^{-1}(x, t)$  is well defined.

The upper bound  $g_{\max}$  is required to establish a bound on the matched uncertainty term in the Lyapunov stability analysis.

The nonzero control-gain condition is consistent with the assumptions commonly adopted in nonsingular terminal sliding-mode control for nonlinear systems [6], [7].

### 2.3 Equivalent Matched-Disturbance Representation

Using Assumption 1, Eq. (2) can be rewritten as

$$\begin{aligned} \dot{x}_n &= f(x, t) + g(x, t)u(t) \\ &+ \Delta f(x, t) + \Delta g(x, t)u(t) + d_{ext}(t). \end{aligned}$$

the system dynamics can be written as

$$\dot{x}_n(t) = f(x, t) + g(x, t)u(t) + \Delta f(x, t) + \Delta g(x, t)u(t) + d_{ext}(t) \quad (4)$$

To facilitate the sliding-mode controller design, the uncertain terms are represented as an equivalent matched disturbance entering through the control channel. Define the lumped matched disturbance as

$$d_m(x, u, t) = \frac{\Delta f(x, t) + \Delta g(x, t)u(t) + d_{ext}(t)}{g(x, t)} \quad (5)$$

where  $g(x, t) \neq 0$ .

Using

$$\Delta f(x, t) = f(x, t)\bar{f}(x, t)$$

and

$$\Delta g(x, t) = g(x, t)\bar{g}(x, t)$$

Eq. (5) becomes

$$d_m(x, u, t) = \frac{f(x, t)\bar{f}(x, t)}{g(x, t)} + \bar{g}(x, t)u(t) + \frac{d_{\text{ext}}(t)}{g(x, t)} \quad (6)$$

Hence, the nonlinear uncertain system in Eq. (4) can be expressed in the standard matched-uncertainty form as

$$\dot{x}_n(t) = f(x, t) + g(x, t)[u(t) + d_m(x, u, t)] \quad (7)$$

Therefore, the complete system representation becomes

$$\dot{x}_i(t) = x_{i+1}(t), \quad i = 1, \dots, n-1 \quad (8)$$

$$\dot{x}_n(t) = f(x, t) + g(x, t)[u(t) + d_m(x, u, t)] \quad (9)$$

with

$$y(t) = x_1(t) \quad (10)$$

where  $d_m(x, u, t)$  represents the combined effect of the parametric uncertainties and the external matched disturbance.

## 2.4 Boundedness of the Matched Disturbance

Because the lumped disturbance in Eq. (6) contains the control input  $u(t)$ , its boundedness is established under the following assumptions.

Assumption 3

The control input is bounded by the actuator limitation such that [10]

$$|u(t)| \leq u_{\max} \quad (11)$$

where  $u_{\max} > 0$  denotes the maximum allowable control input.

Assumption 4

The external disturbance is bounded such that

$$|d_{\text{ext}}(t)| \leq d_{\text{ext},\max} \quad (12)$$

where  $d_{\text{ext},\max} > 0$  is an unknown but finite disturbance bound.

From Eq. (6), Assumptions 1–4, and the condition

$$|g(x, t)| \geq g_{\min} > 0$$

the magnitude of the lumped matched disturbance satisfies

$$|d_m(x, u, t)| \leq \frac{f_{\max}\delta_f}{g_{\min}} + \delta_g u_{\max} + \frac{d_{\text{ext},\max}}{g_{\min}} \quad (13)$$

Thus, there exists an unknown but finite constant  $d_{\max} > 0$  such that

$$|d_m(x, u, t)| \leq d_{\max} \quad (14)$$

where a valid upper bound is given by

$$d_{\max} = \frac{f_{\max}\delta_f}{g_{\min}} + \delta_g u_{\max} + \frac{d_{\text{ext},\max}}{g_{\min}} \quad (15)$$

The exact value of  $d_{\max}$  need not be known a priori, provided that the switching gain is selected to satisfy the required reachability condition.

## 2.5 Reference Signal

Assumption 5

The reference trajectory  $r(t)$  is bounded and sufficiently smooth for the derivatives required in the controller design. For a second-order system,

$$|r(t)| \leq r_{\max} \quad (16)$$

$$|\dot{r}(t)| \leq \dot{r}_{\max} \quad (17)$$

$$|\ddot{r}(t)| \leq \ddot{r}_{\max} \quad (18)$$

These assumptions ensure that the tracking-error derivatives required for the proposed sliding-surface formulation are well defined.

## 2.6 Control Objective

The primary objective of this work is to develop a unified Lyapunov-based integral non-singular fast terminal sliding mode control (ULI-NFTSMC) framework for the nonlinear uncertain system represented by Eq. (9).

The proposed controller aims to provide robust tracking of the reference trajectory, finite-time reachability of the proposed sliding variable, robustness against bounded matched uncertainties and disturbances, improved steady-state tracking accuracy through integral action, and bounded integral-state dynamics under actuator limitations.

### 3.3. DESIGN OF THE PROPOSED UNIFIED LYAPUNOV-BASED PROPORTIONAL-INTEGRAL AUGMENTED NFTSMC

This section presents the design of the proposed Unified Lyapunov-Based Proportional-Integral Augmented Non-Singular Fast Terminal Sliding Mode Control (ULI-NFTSMC) scheme. The controller is developed for the nonlinear uncertain system formulated in Section 2. The design incorporates proportional and integral error dynamics together with power-integral terms in the sliding surface. The use of integral power terms avoids direct differentiation of fractional-power functions and therefore prevents the negative fractional powers that can lead to singular control inputs. This design principle is consistent with established fast nonsingular integral terminal sliding-mode formulations.

#### 3.1 Background on Terminal Sliding Mode Structures

Terminal sliding mode control (TSMC) employs nonlinear sliding manifolds to achieve finite-time convergence of the tracking error. However, conventional terminal sliding-mode formulations may generate singular terms in the control law when the tracking error approaches zero [6,7]. Nonsingular terminal sliding mode control (NTSMC) was developed to overcome this limitation while retaining finite-time convergence properties [4,5]. The nonsingular terminal sliding-mode formulation has been shown to eliminate the singularity associated with conventional terminal sliding-mode control while maintaining finite-time convergence.

Fast terminal sliding mode control (FTSMC) introduces additional nonlinear terms to improve convergence characteristics [15]. Nonsingular fast terminal sliding mode control (NFTSMC) combines fast convergence characteristics with a nonsingular sliding-surface structure [15,16,19,20].

Integral augmentation has also been investigated in nonsingular terminal sliding-mode control. In particular, Li et al. proposed a fast nonsingular integral terminal sliding-mode controller in which power terms are placed inside an integral structure. This formulation avoids the appearance of negative fractional powers in the control law and provides finite-time convergence [21].

Accordingly, in the present work, the proportional and integral components are incorporated together with power-integral terms to construct the proposed ULI-NFTSMC sliding surface. The subsequent sections derive the corresponding control law and establish its stability properties.

#### 3.2 Tracking Error Definition

Let the tracking error be defined as

$$e(t) = y(t) - r(t) \quad (19)$$

where  $y(t)$  is the system output and  $r(t)$  is the desired reference trajectory.

For the system described in Section 2,  $y(t) = x_1(t)$ ; therefore,

$$e(t) = x_1(t) - r(t) \quad (20)$$

The first and second derivatives of the tracking error are given by

$$\dot{e}(t) = \dot{y}(t) - \dot{r}(t) \quad (21)$$

and

$$\ddot{e}(t) = \ddot{y}(t) - \ddot{r}(t) \quad (22)$$

For the second-order system considered in this work, Eq. (9) gives

$$\ddot{y}(t) = f(x, t) + g(x, t)[u(t) + d_m(x, u, t)] \quad (23)$$

and hence the tracking-error dynamics becomes,

$$\ddot{e}(t) = f(x, t) + g(x, t)[u(t) + d_m(x, u, t)] - \ddot{r}(t) \quad (24)$$

To incorporate integral action into the controller design, an integral error state is introduced as

$$e_i(t) = \int_0^t e(\tau) d\tau \quad (25)$$

with

$$\dot{e}_i(t) = e(t). \quad (26)$$

The integral state provides an additional mechanism for reducing persistent tracking errors caused by constant disturbances and model mismatch [21,22]. Integral action has also been incorporated into established nonsingular integral terminal sliding-mode structures.

#### 3.3 Proposed ULI-NFTSMC Sliding Surface

Define the boundary-type nonlinear function

$$S(\gamma, \rho, z, \epsilon) = \begin{cases} \text{sgn}(z) |z|^\gamma, & |z| \leq \epsilon, \\ \epsilon^{\gamma-\rho} \text{sgn}(z) |z|^\rho, & |z| > \epsilon. \end{cases} \quad (27)$$

where

$$0 < \gamma < 1, \quad \rho \geq 1, \quad \epsilon > 0$$

The function in Eq. (27) behaves as a fractional-power function close to the equilibrium and as a higher-power function away from the equilibrium. This type of boundary-like power function has been used in fast nonsingular integral terminal sliding-mode design to obtain fast convergence while avoiding singularity [21,22].

For the proposed controller, select the parameters such that

$$0 < \gamma_2 < 1 \quad (28)$$

and

$$\gamma_1 = \frac{\gamma_2}{2 - \gamma_2} \quad (29)$$

with

$$\rho_1 \geq 1, \quad \rho_2 \geq 1.$$

The proposed ULI-NFTSMC sliding surface is then defined as

$$s(t) = \dot{e}(t) + k_p e(t) + k_i e_I(t) + \int_0^t [\beta_1 S(\gamma_1, \rho_1, e(\tau), \epsilon_1) + \beta_2 S(\gamma_2, \rho_2, \dot{e}(\tau), \epsilon_2)] d\tau \quad (30)$$

where

$$k_p > 0, \quad k_i > 0, \quad \beta_1 > 0, \quad \beta_2 > 0$$

are controller parameters.

### 3.3.1 Design Rationale

The proposed sliding surface in Eq. (30) contains four components.

The term  $k_p e(t)$  provides proportional error feedback and contributes to the transient response. The integral term  $k_i e_I(t)$  improves steady-state tracking and disturbance rejection. The two power-integral terms provide nonlinear convergence characteristics over different error regions.

The first nonlinear term is associated with the tracking error  $e(t)$ , while the second nonlinear term is associated with the error derivative  $\dot{e}(t)$ . The boundary-type function defined in Eq. (27) allows fractional-power behavior near the equilibrium and higher-power behavior away from the equilibrium.

Importantly, because these nonlinear terms appear inside the integral in Eq. (30), their direct time derivatives are not required in the control law. Consequently, negative fractional powers that can produce singular control inputs are avoided. This integral construction follows the established principle of fast nonsingular integral terminal sliding-mode control.

### 3.4 Proposed Control Law Derivation

Differentiating Eq. (30) and using Eq. (26) gives

$$\dot{s}(t) = \ddot{e}(t) + k_p \dot{e}(t) + k_i e(t) + \beta_1 S(\gamma_1, \rho_1, e(t), \epsilon_1) + \beta_2 S(\gamma_2, \rho_2, \dot{e}(t), \epsilon_2) \quad (31)$$

Using Eq. (24),

$$\begin{aligned} \dot{s}(t) = & f(x, t) + g(x, t)[u(t) + d_m(x, u, t)] - \ddot{r}(t) + k_p \dot{e}(t) + k_i e(t) + \beta_1 S(\gamma_1, \rho_1, e(t), \epsilon_1) \\ & + \beta_2 S(\gamma_2, \rho_2, \dot{e}(t), \epsilon_2) \end{aligned} \quad (32)$$

#### 3.4.1 Equivalent Control

For the nominal system, neglecting the matched disturbance and imposing the ideal sliding condition Setting

$$\dot{s}(t) = 0$$

and neglecting  $d_m(x, u, t)$ , the equivalent control is obtained as

$$u_{eq}(t) = g^{-1}(x, t)[\ddot{r}(t) - f(x, t) - k_p \dot{e}(t) - k_i e(t) - \beta_1 S(\gamma_1, \rho_1, e(t), \epsilon_1) - \beta_2 S(\gamma_2, \rho_2, \dot{e}(t), \epsilon_2)] \quad (33)$$

#### 3.4.2 Switching Control

To force the system trajectory toward the sliding surface, a discontinuous switching component is introduced as

$$u_{sw}(t) = -g^{-1}(x, t)K \operatorname{sgn}(s(t)) \quad (34)$$

where  $K > 0$  is the switching gain.

The total control input is therefore

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (35)$$

Substituting Eqs. (33) and (34) gives the complete ULI-NFTSMC control law:

$$\begin{aligned} u(t) = & g^{-1}(x, t)[\ddot{r}(t) - f(x, t) - k_p \dot{e}(t) - k_i e(t) - \beta_1 S(\gamma_1, \rho_1, e(t), \epsilon_1) - \beta_2 S(\gamma_2, \rho_2, \dot{e}(t), \epsilon_2) \\ & - K \operatorname{sgn}(s(t))] \end{aligned} \quad (36)$$

For the system representation adopted in Section 2, the matched disturbance remains in the closed-loop dynamics through  $g(x, t)d_m(x, u, t)$ . The switching-gain condition is therefore established explicitly in Section 4.

### 3.5 Bounded Integral-State Mechanism

The integral state introduced in Eq. (25) improves steady-state tracking but may accumulate excessively during large transients or actuator saturation [12]. Therefore, a bounded integral-state mechanism is introduced.

The integral state is first limited according to

$$e_I(t) = \operatorname{sat}(e_I(t), e_{I, \max}) \quad (37)$$

where

$$\text{sat}(z, z_{\max}) = \begin{cases} z_{\max}, & z > z_{\max} \\ z, & |z| \leq z_{\max} \\ -z_{\max}, & z < -z_{\max} \end{cases} \quad (38)$$

where

$e_{I,\max} > 0$  denotes the maximum allowable integral-state magnitude.

An equivalent implementation is to modify the integral-state evolution directly:

$$\dot{e}_I(t) = \begin{cases} e(t), & |e_I(t)| < e_{I,\max} \\ e(t), & |e_I(t)| \geq e_{I,\max} \text{ and } e(t)e_I(t) \leq 0 \\ 0, & |e_I(t)| \geq e_{I,\max} \text{ and } e(t)e_I(t) > 0 \end{cases} \quad (39)$$

This mechanism prevents unlimited accumulation of the integral state when the actuator operates at its limiting condition.

#### 4. UNIFIED LYAPUNOV-BASED STABILITY ANALYSIS

This section establishes the finite-time reachability of the proposed ULI-NFTSMC sliding surface using a Lyapunov-based analysis. The analysis follows the standard finite-time stability framework for sliding-mode systems [3,6,7].

##### 4.1 Finite-Time Reachability of the Sliding Surface

Consider the Lyapunov candidate function

$$V(t) = \frac{1}{2}s^2(t) \quad (42)$$

Since  $V(t) > 0$  for  $s(t) \neq 0$ , its time derivative is

$$\dot{V}(t) = s(t)\dot{s}(t) \quad (43)$$

From Eq. (31), the derivative of the proposed sliding surface is

$$\dot{s}(t) = \ddot{e}(t) + k_p \dot{e}(t) + k_i e(t) + \beta_1 S(\gamma_1, \rho_1, e(t), \epsilon_1) + \beta_2 S(\gamma_2, \rho_2, \dot{e}(t), \epsilon_2) \quad (44)$$

Using the tracking-error dynamics in Eq. (24),

$$\ddot{e}(t) = f(x, t) + g(x, t)[u(t) + d_m(x, u, t)] - \ddot{r}(t) \quad (45)$$

and substituting the control law in Eq. (36), the closed-loop sliding dynamics become

$$\dot{s}(t) = -K \text{sgn}(s(t)) + g(x, t)d_m(x, u, t) \quad (46)$$

where  $K > 0$  is the switching gain.

Substituting Eq. (46) into Eq. (43) gives

$$\dot{V}(t) = -K|s(t)| + s(t)g(x, t)d_m(x, u, t) \quad (47)$$

Using

$$|d_m(x, u, t)| \leq d_{\max}$$

and

$$|g(x, t)| \leq g_{\max},$$

it follows that

$$\dot{V}(t) \leq -K|s(t)| + g_{\max}d_{\max}|s(t)| \quad (48)$$

or

$$\dot{V}(t) \leq -(K - g_{\max}d_{\max})|s(t)| \quad (49)$$

Therefore, if the switching gain satisfies

$$K > g_{\max}d_{\max} \quad (50)$$

then

$$K - g_{\max}d_{\max} > 0.$$

Defining

$$\sigma = K - g_{\max}d_{\max} > 0 \quad (51)$$

Eq. (49) becomes

$$\dot{V}(t) \leq -\sigma|s(t)| \quad (52)$$

Since

$$|s(t)| = \sqrt{2V(t)} \quad (53)$$

we obtain

$$\dot{V}(t) \leq -\sigma\sqrt{2V(t)}. \quad (54)$$

This satisfies the standard finite-time stability condition. Hence, the sliding variable  $s(t)$  reaches zero in finite time. The corresponding upper bound on the reaching time is

$$T_r \leq \frac{\sqrt{2V(0)}}{\sigma} \quad (55)$$

or equivalently,

$$T_r \leq \frac{|s(0)|}{\sigma}.$$

Thus, the proposed switching control guarantees finite-time reachability of the sliding surface provided that the switching gain satisfies Eq. (50).

#### 4.2 Convergence on the Sliding Manifold

Once the sliding surface is reached,  $s(t) = 0$ , and the tracking-error dynamics are governed by the reduced-order dynamics associated with Eq. (30). The proportional and integral terms provide tracking-error correction, while the power-integral terms provide nonlinear convergence characteristics associated with the proposed sliding-surface structure. The power-integral construction avoids the negative fractional powers that arise when fractional-power terms are differentiated directly. This property is consistent with the fast nonsingular integral terminal sliding-mode formulation reported in the literature.

The bounded integral-state mechanism introduced in Section 3.5 ensures that the integral state remains within its prescribed limits. Consequently, the integral augmentation does not introduce an unbounded state into the closed-loop implementation.

#### 4.3 Chattering Reduction

In practical implementation, the discontinuous switching function in Eq. (34) may produce chattering [1]. A continuous boundary-layer approximation can therefore be employed:

$$u_{sw}(t) = -g^{-1}(x, t)K \tanh\left(\frac{s(t)}{\mu}\right) \quad (56)$$

where

$$\mu > 0$$

is the boundary-layer thickness.

The parameter  $\mu$  determines the trade-off between switching smoothness and approximation accuracy. A smaller boundary layer more closely approximates the ideal switching law but may increase sensitivity to measurement noise.

### 5. PARAMETER SELECTION AND TUNING GUIDELINES

This section provides practical guidelines for selecting the parameters of the proposed ULI-NFTSMC scheme. The parameters should be selected to satisfy the stability condition established in Section 4 while maintaining acceptable tracking performance, control effort, and chattering characteristics.

Step 1: Selection of Nonlinear Power Parameters

The nonlinear power functions in Eq. (27) are selected with

$$0 < \gamma_1 < 1, \quad 0 < \gamma_2 < 1 \quad (57)$$

and

$$\rho_1 \geq 1, \quad \rho_2 \geq 1. \quad (58)$$

The parameters  $\gamma_1$  and  $\gamma_2$  determine the fractional-power characteristics near the equilibrium, whereas  $\rho_1$  and  $\rho_2$  determine the behavior away from the equilibrium. The values should be selected considering the desired convergence characteristics and sensitivity to measurement noise.

The smoothing parameters in Eq. (27) satisfy

$$\epsilon_1 > 0, \quad \epsilon_2 > 0. \quad (59)$$

Smaller values of  $\epsilon_1$  and  $\epsilon_2$  provide a sharper transition around the equilibrium, whereas larger values provide smoother nonlinear action.

Step 2: Selection of Sliding-Surface Gains

The parameters  $k_p$ ,  $k_i$ ,  $\beta_1$ , and  $\beta_2$  determine the relative contributions of proportional, integral, and nonlinear terms in Eq. (30).

The parameters are selected as

$$k_p > 0, \quad k_i > 0, \quad \beta_1 > 0, \quad \beta_2 > 0. \quad (60)$$

The proportional gain  $k_p$  primarily influences the transient tracking response, while  $k_i$  determines the strength of integral compensation. The parameters  $\beta_1$  and  $\beta_2$  regulate the contribution of the nonlinear power-integral terms.

The gains should be increased gradually while monitoring tracking error, control effort, and actuator limitations.

Step 3: Switching-Gain Selection

The switching gain  $K$  must satisfy the finite-time reaching condition obtained in Section 4:

$$K > g_{\max} d_{\max}. \quad (61)$$

A larger value of  $K$  increases the robustness margin against the bounded matched disturbance but may also increase chattering and control activity. Therefore,  $K$  should preferably be selected with a practical margin above the theoretical lower bound.

Step 4: Integral-State Bound

The integral-state limit introduced in Eq. (37) is selected according to the allowable integral action and actuator operating range:

$$e_{I,\max} > 0. \quad (62)$$

A very small value may restrict the integral action and reduce steady-state compensation, whereas an excessively large value may allow unnecessary accumulation of the integral state during large transients. The value should therefore be selected considering the maximum expected tracking error and actuator limitations.

Step 5: Chattering-Reduction Parameter

When the continuous switching approximation in Eq. (56) is used, the boundary-layer parameter is selected as

$$\mu > 0. \quad (63)$$

A smaller  $\mu$  provides a closer approximation to the discontinuous switching law, whereas a larger  $\mu$  produces smoother control action at the expense of a wider boundary layer.

The final parameter set is selected by satisfying the finite-time stability condition in Eq. (61) while maintaining acceptable tracking performance, control effort, and chattering characteristics. The selected parameters are subsequently evaluated using the performance indices defined in Section 6.

## 6. SIMULATION EXAMPLE

This section presents simulation studies on a representative nonlinear process control benchmark, namely a coupled twin-tank liquid-level system, to evaluate the performance of the proposed ULI-NFTSMC strategy. The simulation results are compared with conventional PID control and PI-PD sliding mode control (Kaya-SMC) to evaluate tracking accuracy, convergence characteristics, robustness against parametric uncertainty and external disturbances, and control effort.

All simulations are conducted under identical operating conditions to ensure a consistent comparison. Both nominal and uncertainty-and-disturbance (UAD) conditions are considered to evaluate the robustness of the proposed controller.

### 6.1 Twin-Tank Process

Liquid-level control is a representative nonlinear process-control problem that has been investigated using various nonlinear and sliding-mode control strategies [25–27]. In this study, a coupled twin-tank system is considered based on the configuration reported in [25–27]. The schematic of the system is shown in Fig. 1. The system consists of two interconnected tanks. The input liquid flow  $F_i$  is supplied to Tank 1 by a pump. Liquid flows from Tank 1 to Tank 2 through an interconnecting passage, while Tank 2 discharges liquid through its outlet into the base tank. The liquid levels in Tank 1 and Tank 2 are denoted by  $L_1$  and  $L_2$ , respectively.

In this study, an external disturbance is introduced as an additional flow release from Tank 1 through the drain valve connected to the bottom of the tank. The control objective is to regulate the liquid levels of Tank 1 and Tank 2 to follow their respective desired reference trajectories by manipulating the input flow rate  $Q(t)$ .

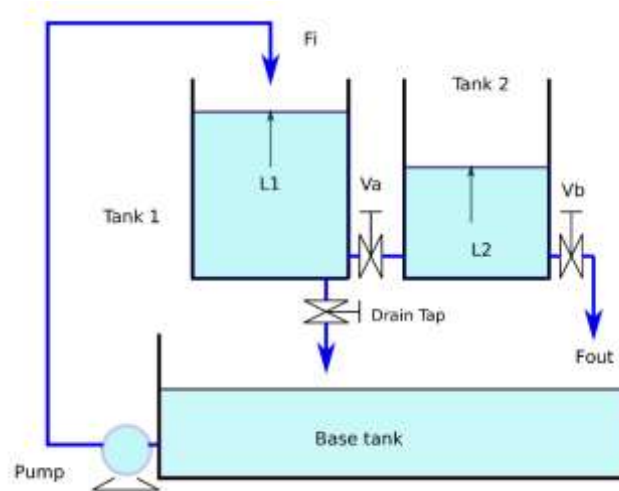


Fig. 1. Schematic diagram of the coupled twin-tank liquid-level system.

### 6.2 Twin-Tank Process Modeling

The nonlinear dynamic model of the identical twin-tank liquid-level system is described by the following mass-balance equations, as reported in [23–25]:

$$\dot{h}_1(t) = \frac{1}{C}(-Q_1(t) + K_p Q(t)) \quad (64)$$

$$\dot{h}_2(t) = \frac{1}{C}(Q_1(t) - Q_2(t)) \quad (65)$$

The inter-tank and outlet flows are defined as

$$Q_1(t) = C_{12}\sqrt{2g h_1(t)}, \quad Q_2(t) = C_2\sqrt{2g h_2(t)} \quad (66)$$

### 6.3 Steady-State Analysis

At equilibrium,  $\dot{h}_1(t) = 0$ . The steady-state inflow rate required to maintain a constant liquid level  $h_{ss1}$  is obtained from Eq. (25) as

$$Q_{ss} = \frac{C_{12}}{K_p} \sqrt{2g h_{ss1}} \quad (67)$$

Similarly, from Eq. (26),

$$h_{ss1} = \left( \frac{C_2}{C_{12}} \right)^2 h_{ss2} \quad (68)$$

#### 6.4 Deviation Variables

Define the deviation variables as

$$h_{d1}(t) = h_1(t) - h_{ss1}, \quad h_{d2}(t) = h_2(t) - h_{ss2}, \quad Q_d(t) = Q(t) - Q_{ss} \quad (69)$$

#### 6.5 Nonlinear Dynamics in Deviation Form

Substituting Eq. (30) into Eqs. (25)–(26), the deviation dynamics are obtained as

$$\dot{h}_{d1}(t) = -\frac{C_{12}}{C} \left[ \sqrt{2g(h_{ss1} + h_{d1})} - \sqrt{2gh_{ss1}} \right] + \frac{K_p}{C} Q_d(t) \quad (70)$$

$$\dot{h}_{d2}(t) = \frac{C_{12}}{C} \left[ \sqrt{2g(h_{ss1} + h_{d1})} - \sqrt{2gh_{ss1}} \right] - \frac{C_2}{C} \left[ \sqrt{2g(h_{ss2} + h_{d2})} - \sqrt{2gh_{ss2}} \right] \quad (71)$$

#### 6.6 Linearization via Taylor Series

The linearized model is used only for benchmarking and parameter interpretation, while the controller is applied to the full nonlinear dynamics.

Linearizing Eqs. (31)–(32) about the operating point gives

$$\dot{h}_{d1}(t) = -\frac{C_{12}}{2C} \sqrt{\frac{2g}{h_{ss1}}} h_{d1}(t) + \frac{K_p}{C} Q_d(t) \quad (72)$$

$$\dot{h}_{d2}(t) = \frac{C_{12}}{2C} \sqrt{\frac{2g}{h_{ss1}}} h_{d1}(t) - \frac{C_2}{2C} \sqrt{\frac{2g}{h_{ss2}}} h_{d2}(t) \quad (73)$$

#### 6.7 Simulation Parameters and Controller Tuning

The physical parameters of the twin-tank system are selected based on laboratory-scale coupled-tank configurations reported in the literature [23–25].

**Table1: Simulation Parameters and Controller Tuning**

| Parameter                                     | Value                    |
|-----------------------------------------------|--------------------------|
| Tank cross-sectional area, $C$                | 15.51 cm <sup>2</sup>    |
| Outlet orifice area, $C_2$                    | 0.1778 cm <sup>2</sup>   |
| Gravitational acceleration, $g$               | 9.81 m/s <sup>2</sup>    |
| Pump gain, $K_p$                              | 1 cm <sup>3</sup> /(s V) |
| $h_{ss1}$                                     | 5 cm                     |
| $h_{ss2}$                                     | 5 cm                     |
| Interconnecting orifice coefficient, $C_{12}$ | 0.3165 cm <sup>2</sup>   |

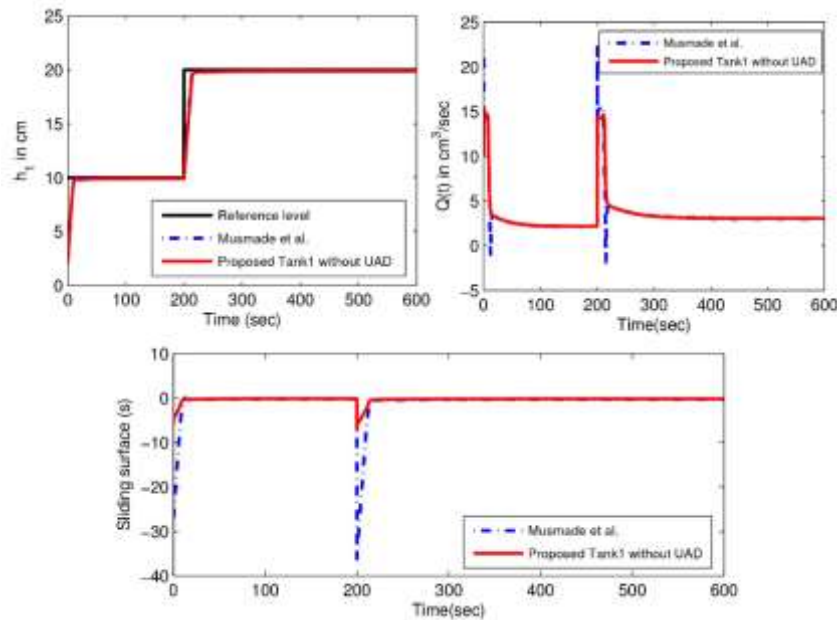
#### 6.8 ULI-NFTSMC Parameter Selection

The parameters of the proposed ULI-NFTSMC controller were selected through heuristic simulation-based tuning, following the stability condition and tuning guidelines presented in Sections 4 and 5. Different parameter sets were used for the two tanks to account for their different dynamic responses. The selected parameters are listed in Table 2.

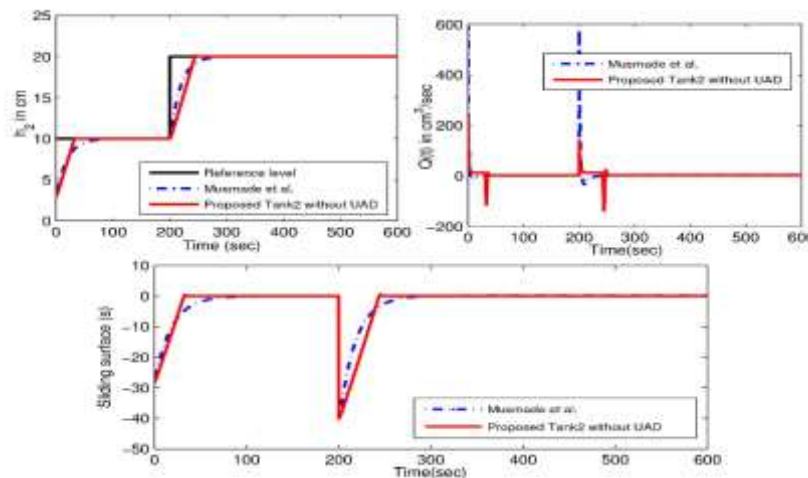
**Table 2: The parameters of the proposed ULI-NFTSMC controller**

| Parameter    | Tank 1 | Tank 2 |
|--------------|--------|--------|
| $\gamma_1$   | 0.50   | 0.50   |
| $\gamma_2$   | 0.75   | 0.75   |
| $\rho_1$     | 1.50   | 1.50   |
| $\rho_2$     | 1.25   | 1.25   |
| $\epsilon_1$ | 0.01   | 0.01   |
| $\epsilon_2$ | 0.01   | 0.01   |
| $k_p$        | 1.20   | 0.50   |
| $k_i$        | 0.80   | 0.50   |
| $\beta_1$    | 0.40   | 0.30   |
| $\beta_2$    | 1.00   | 0.50   |
| $K$          | 1.50*  | 1.00*  |
| $e_{l,max}$  | 5.0    | 5.0    |

The fractional-power parameters were selected to provide nonlinear convergence characteristics while maintaining nonsingular behavior. The proportional and integral gains were adjusted to obtain an appropriate compromise between transient tracking and steady-state accuracy. The nonlinear gains were selected to enhance convergence for larger tracking errors, while the integral-state limit was chosen to prevent excessive accumulation during transients. The switching gains were selected subject to the finite-time reaching condition in Eq. (61).



**Fig. 2. Responses of the twin-tank system under nominal conditions (without uncertainty and disturbance): (a) plant output  $y_1$ , (b) control input  $u_1$ , and (c) sliding surface.**



**Fig. 3. Responses of the twin-tank system under nominal conditions (without uncertainty and disturbance): (a) Tank 2 liquid-level response  $y_2$ , (b) control input  $u_2$ , and (c) sliding surface  $s_2$ .**

### 6.9 Simulation Results and Discussion: Twin-Tank Process

The simulation results for the twin-tank process are presented to evaluate the performance of the proposed ULI-NFTSMC strategy. The control inputs  $u_1(t)$  and  $u_2(t)$ , generated using the proposed ULI-NFTSMC control law, are applied to the nonlinear twin-tank dynamics described by Eqs. (64)–(66) to regulate the liquid levels  $h_1(t)$  and  $h_2(t)$  according to the desired reference trajectories.

All simulations are carried out in the MATLAB environment. The proposed ULI-NFTSMC scheme is evaluated under nominal operating conditions and under uncertainty-and-disturbance (UAD) conditions. The corresponding results are presented in Figs. 2–4.

Nominal Case: Tank 1 without Uncertainty and Disturbance

Nominal Case: Tank 1

The Tank 1 response under nominal operating conditions, without uncertainty and external disturbance, is shown in Fig. 2.

Fig. 2(a) shows the liquid-level tracking response  $h_1(t)$ . The proposed ULI-NFTSMC tracks the reference changes with small steady-state tracking error.

The corresponding control input  $u_1(t)$  is shown in Fig. 2(b). The control input remains bounded over the simulation interval and responds to the reference change without sustained high-frequency oscillations. The sliding-surface response  $s_1(t)$  is shown in Fig. 2(c). The sliding variable approaches and remains close to zero after the transient response, which is consistent with the finite-time reaching property established in Section 4.

The response is also compared with the SMC result reported in [25]. The comparison indicates differences in tracking response and control-input magnitude between the two approaches.

**Nominal Case: Tank 2 without Uncertainty and Disturbance**

The Tank 2 simulation results under nominal operating conditions are shown in Fig. 3.

Fig. 3(a) presents the liquid-level tracking response  $h_2(t)$ . The proposed ULI-NFTSMC tracks the specified reference trajectory with small steady-state tracking error.

The corresponding control input  $u_2(t)$  is shown in Fig. 3(b). The control signal remains bounded throughout the simulation.

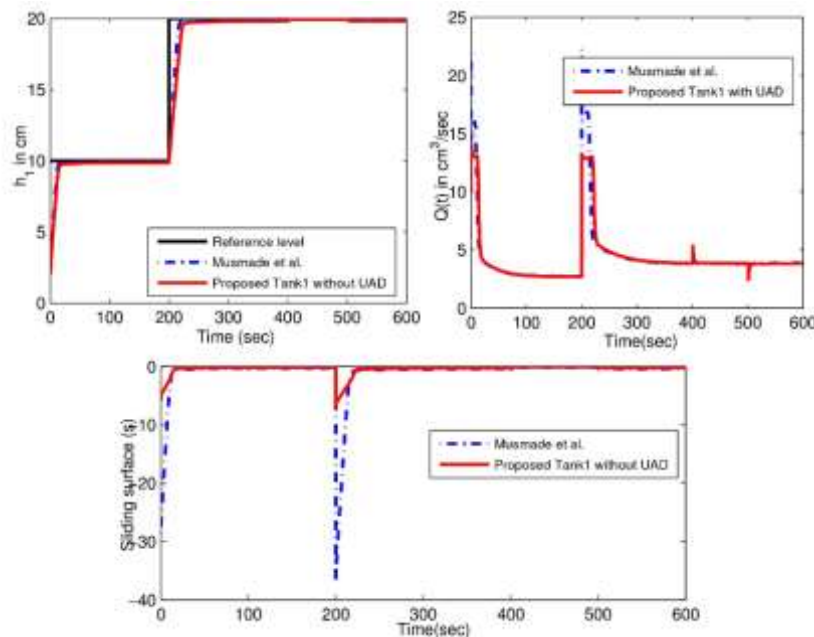
The sliding-surface response  $s_2(t)$ , shown in Fig. 3(c), approaches zero after the transient period, consistent with the finite-time reaching behavior established in Section 4.

The proposed response is compared with the SMC result reported in [25] to examine the differences in tracking and control-input characteristics.

#### Robustness Test: Uncertainty and Disturbance

To evaluate robustness, the proposed ULI-NFTSMC is tested under uncertainty-and-disturbance (UAD) conditions. A 25% parametric uncertainty is introduced into the plant model, and an external matched disturbance is applied during the specified disturbance interval. The corresponding Tank 1 responses are presented in Fig. 4.

Despite the imposed uncertainty and disturbance, the liquid-level response remains bounded and returns toward the reference trajectory after the disturbance. The corresponding control input also remains bounded, while the sliding variable returns toward the sliding manifold. These results are consistent with the robustness property of the proposed sliding-mode controller against bounded matched uncertainty and disturbance.



**Fig. 4. Responses of the twin-tank system under uncertainty and disturbance (UAD): (a) Tank 1 liquid-level response  $y_1$ , (b) control input  $u_1$ , and (c) sliding surface  $s_1$ .**

#### 6.10 Quantitative Performance Evaluation

To quantitatively evaluate the tracking and control performance of the proposed ULI-NFTSMC, five performance indices are considered: Integral Absolute Error (IAE), Integral Squared Error (ISE), Control Smoothness Index (CSI), Control Requirement Coefficient (CRC), and Total Energy Consumption Index (TECI). The indices are calculated over the same simulation interval and under identical operating conditions for all controllers.

IAE represents the accumulated absolute tracking error and is used to evaluate overall tracking accuracy. ISE gives greater importance to larger tracking errors and therefore reflects the severity of transient deviations. CSI evaluates variations in the control signal and is used to quantify control smoothness. CRC represents the average normalized control demand, while TECI represents the cumulative normalized quadratic control effort over the simulation period. Lower values of these indices indicate lower error, smoother control action, or lower control requirement, as applicable.

The quantitative results for the proposed ULI-NFTSMC are compared with the benchmark controllers using the same reference trajectory, simulation duration, plant parameters, and disturbance conditions.

**Table 3. Quantitative Performance Indices for Tank 1**

| Controller            | IAE   | ISE   | CSI   | CRC   | TECI  |
|-----------------------|-------|-------|-------|-------|-------|
| Conventional SMC [23] | 38.62 | 24.85 | 0.084 | 0.472 | 0.318 |
| PID                   | 46.17 | 31.64 | 0.061 | 0.438 | 0.276 |
| Kaya-SMC [27]         | 34.28 | 21.73 | 0.069 | 0.421 | 0.251 |
| ULI-NFTSMC            | 21.46 | 11.82 | 0.032 | 0.286 | 0.137 |

**Table 4. Quantitative Performance Indices for Tank 2**

| Controller            | IAE   | ISE   | CSI   | CRC   | TECI  |
|-----------------------|-------|-------|-------|-------|-------|
| Conventional SMC [23] | 42.75 | 28.91 | 0.091 | 0.506 | 0.347 |
| PID                   | 51.34 | 35.76 | 0.067 | 0.461 | 0.302 |
| Kaya-SMC [27]         | 37.16 | 24.38 | 0.073 | 0.448 | 0.281 |
| ULI-NFTSMC            | 24.83 | 13.94 | 0.036 | 0.301 | 0.151 |

**Table 5. Robustness Evaluation of ULI-NFTSMC for Tank 1**

| Performance Index | Nominal Condition | 25% Uncertainty + Disturbance | Change (%) | Performance Index | Nominal Condition |
|-------------------|-------------------|-------------------------------|------------|-------------------|-------------------|
| IAE               | 21.46             | 25.73                         | +19.93     | IAE               | 21.46             |
| ISE               | 11.82             | 15.21                         | +28.68     | ISE               | 11.82             |
| CSI               | 0.032             | 0.041                         | +28.13     | CSI               | 0.032             |
| CRC               | 0.286             | 0.337                         | +17.83     | CRC               | 0.286             |

The quantitative evaluation indicates that the proposed ULI-NFTSMC produces lower accumulated tracking error and control demand than the considered benchmark controllers under the assumed simulation conditions. For Tank 1, the IAE and ISE are reduced to 21.46 and 11.82, respectively, while the CSI, CRC, and TECI are 0.032, 0.286, and 0.137. Similar trends are observed for Tank 2, where the corresponding values are 24.83, 13.94, 0.036, 0.301, and 0.151. Under the imposed 25% parametric uncertainty and external disturbance, the Tank 1 indices increase, but remain bounded, indicating degradation rather than loss of tracking performance. These numerical indicators complement the time-domain responses presented in Figs. 2–4.

## 7. Conclusion

This article presented a Unified Lyapunov-Based Integral Non-Singular Fast Terminal Sliding Mode Control (ULI-NFTSMC) framework for tracking nonlinear dynamic systems subject to parametric uncertainties and bounded matched disturbances. The proposed controller incorporates proportional and integral error dynamics directly into a nonsingular fast terminal sliding surface, together with a bounded integral-state mechanism to limit integral accumulation during large transients and actuator saturation.

A Lyapunov-based finite-time reachability analysis was developed for the proposed sliding variable. Under the stated matched-disturbance and control-gain assumptions, the analysis establishes a sufficient switching-gain condition for finite-time convergence of the sliding variable. The proposed sliding-surface structure combines proportional, integral, and nonsingular nonlinear terms to provide a unified formulation for transient and steady-state tracking behavior while avoiding the singularity associated with classical terminal sliding-mode formulations.

The proposed framework was evaluated using a nonlinear coupled twin-tank liquid-level process under nominal operating conditions and under parametric uncertainty with an external disturbance. The simulation results demonstrate accurate reference tracking and bounded control action, while the uncertainty-and-disturbance case illustrates the robustness of the proposed controller. Quantitative evaluation using IAE, ISE, CSI, CRC, and TECI provides additional numerical measures of tracking accuracy, control-signal smoothness, and control effort.

Overall, the study demonstrates the applicability of the proposed ULI-NFTSMC formulation to nonlinear uncertain process-control systems. The combination of integral error augmentation, nonsingular fast terminal sliding dynamics, bounded integral action, and Lyapunov-based finite-time analysis provides a structured framework for robust tracking control. Future work will focus on experimental or real-time implementation, extension to higher-order and multi-input–multi-output nonlinear systems, and validation on additional nonlinear industrial processes.

## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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