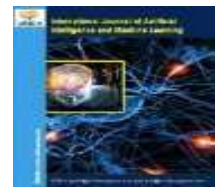




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Research Paper

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Evaluating AI, Traditional, and Hybrid Learning Effectiveness: A Compartmental Model with APOS-Defined Mastery

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Abstract

This paper extends the Dynamic Peer Learning Evaluation (DPLE) model, it is a refinement of the Traditional versus Peer Learning (TPL) model, along three structural dimensions. First, the peer-learning pathway is redefined as an AI-assisted learning pathway. This reflects the growing role of conversational and adaptive AI tutoring systems in providing on-demand, dialogic support once offered only by human peers. Second, a hybrid compartment is introduced to represent students who use both traditional instruction and AI-assisted learning at the same time. Under a strict two-pathway structure, these students are indistinguishable from single-pathway learners. Third, the Mastery compartment is redefined using APOS theory (Action, Process, Object, Schema). Under this definition, mastery is achieved only when a student reaches the Schema level of conceptual understanding, not by an unspecified notion of curriculum completion. APOS is used strictly as a measurement definition for the Mastery state. It is not modelled as an internal transition sequence within the ODE system. The resulting model has five compartments: Unengaged (U), Traditional learners (T), AI-assisted learners (A), Hybrid learners (H), and Mastery (M). It is governed by a system of nonlinear ordinary differential equations with interaction-driven recruitment and hybridisation terms. The model also introduces the Relative Mastery Efficiency (RME), a normalised index that expresses each pathway's mastery transition rate as a proportion of the fastest-performing pathway. This allows traditional, AI-assisted, and hybrid learning to be ranked together. A seven-step application procedure is also provided for practitioners. It extends the DPLE procedure with an APOS-based mastery-assessment step. As with the TPL and DPLE models, the DHLE model is presented here as a fully developed theoretical framework, ahead of empirical validation, which is now ready to proceed using longitudinal classroom data.

Keywords: AI-assisted learning; hybrid learning; traditional learning; compartmental model; ordinary differential equations; APOS theory; mastery.

1. Introduction

The comparative evaluation of traditional, instructor-led teaching and collaborative, learner-driven approaches remains a central question in educational research. Mazibuko (2026) formalised this comparison through the Traditional versus Peer Learning (TPL) model, a compartmental framework classifying students into skilled and unskilled sub-groups within each learning modality. Mazibuko (2026b) subsequently developed the Dynamic Peer Learning Evaluation (DPLE) model, which replaced the TPL model's static, linear transition structure with interaction-driven, nonlinear recruitment dynamics modelled on the Susceptible–Infected–Recovered (SIR) framework of Kermack and McKendrick (1927), and consolidated learning outcomes into a single unified Mastery compartment.

The DPLE model represents a substantial advance over the TPL model, yet three structural gaps remain relevant to how peer-style learning support is now delivered and experienced in classrooms. This paper addresses each in turn.

(i) The changing identity of the “peer.” The DPLE model, following its predecessor, treats peer learning as collaboration among fellow students. However, the rapid uptake of conversational and adaptive artificial intelligence (AI) tutoring systems has introduced a new source of always-available, dialogic instructional support that plays a functionally similar role to a human peer: it responds to a learner's specific query, offers explanations at the learner's pace, and scaffolds understanding without the formal authority of an instructor (Kasneci et al., 2023). This mirrors the kind of assisted performance described in socio-cultural accounts of learning support (Vygotsky, 1978), except that the “more capable other” is now, in many classrooms, an AI system rather than a classmate. This motivates redefining the peer-learning pathway of the DPLE model as an AI-assisted learning pathway.

(ii) The absence of a hybrid pathway. Both the TPL and DPLE models assume that a student is engaged through exactly one learning modality at a time. In practice, however, many students combine instructor-led instruction with AI-assisted support. For example, a student may attend lectures and subsequently use an AI tutor to clarify or extend the material, relying equally on traditional and AI-assisted learning approaches. A two-pathway model cannot represent this substantial and pedagogically distinct population,

nor can it capture whether combining pathways accelerates or dilutes progress toward mastery. This gap is analogous to the treatment of co-exposure

or dual-pathway transmission in extended epidemiological compartmental models, and motivates the introduction of an explicit Hybrid compartment.

(iii) The undefined construct of “mastery.” The DPLE model consolidates achievement into a single Mastery compartment but defines entry into it only as having “achieved the predetermined curriculum objectives,” without specifying what constitutes achievement in cognitive or pedagogical terms. This paper adopts APOS theory (Dubinsky, 1991; Asiala et al., 1996; Arnon et al., 2014), a well-established constructivist account of how learners build mathematical and scientific understanding through the stages of Action, Process, Object, and Schema, to give the Mastery compartment a concrete, assessable definition: a student is counted as having achieved mastery only when assessment evidence places them at the Schema level. Consistent with the scope of this paper, APOS is used only to define the mastery criterion; it is not used to model additional internal transitions.

The resulting model, referred to here as the Dynamic Hybrid Learning Evaluation (DHLE) model, retains the mathematical tractability and interaction-driven philosophy of the DPLE model while extending it to a five-compartment, nonlinear ODE system suited to contemporary, AI-mediated classroom environments.

2. Literature Review

2.1 From Peer Learning to AI-Assisted Learning

Comparative studies of peer and traditional instruction report mixed findings. Sabaq, Farouk, and Ismail (2016) found significantly higher knowledge and performance scores among nursing students similarly reported higher performance and increased peer interaction relative to traditional instruction. Conversely, Schwerdt and Wuppermann (2009) found traditional teaching associated with higher achievement in a within-student, between-subject design, and Tularam (2018) noted that improvements attributed to non-traditional methods in mathematics and engineering education are often marginal. Wang (2022) concluded that the appropriate choice of method depends on instructional context, student needs, and learning objectives rather than on a universal ranking of methods.

A more recent development complicates this comparison further: the peer role in collaborative learning is increasingly filled, in part or in whole, by conversational AI systems. Kasneci et al. (2023) discuss how large language models can provide on-demand explanation, worked examples, and Socratic-style questioning that closely resembles the support a knowledgeable peer would offer, while remaining available outside scheduled class time and without the social risk some students associate with asking classmates for help. Framed in socio-cultural terms, an AI tutor can function as a “more capable other” within a learner’s zone of proximal development (Vygotsky, 1978), providing the scaffolding historically associated with peer or instructor support. This literature motivates treating AI-assisted learning as a distinct, first-class instructional pathway in a compartmental evaluation model, rather than as a subtype of either traditional or peer learning.

2.2. Hybrid and Blended Learning Environments

Blended learning research has long argued that combining modalities can outperform either modality in isolation when the combination is deliberately designed rather than incidental (Garrison & Kanuka, 2004). This literature supports the view that students who draw on both instructor-led and AI-assisted support form a pedagogically distinct group, whose trajectory toward mastery need not equal the average, or even the sum, of the single-pathway trajectories. Existing compartmental education models, including the TPL and DPLE models, do not accommodate this population explicitly, since they assume mutually exclusive modality membership.

2.3. Mathematical Compartmental Models in Education

Compartmental modelling in education is rooted in epidemiological theory, particularly the Susceptible–Infected–Recovered (SIR) model of Kermack and McKendrick (1927), in which a population is divided into susceptible (S), infected (I), and recovered (R) groups governed by a transmission rate β and a recovery rate γ . Mazibuko and Maharaj (2024a) adapted this structure to evaluate higher-order thinking skills through the five-compartment SVHIR model, and Mazibuko and Maharaj (2024b) extended it further through the RCUSP model for general curriculum evaluation. Building on this lineage, the TPL model (Mazibuko, 2026) applied compartmental principles directly to the comparison of peer and traditional learning using four compartments and linear ODEs, and the DPLE model (Mazibuko, 2026b) replaced these linear terms with interaction-driven, nonlinear recruitment dynamics (α_{UT} and θ_{UP}) analogous to the β_{SI} term of the SIR model.

Epidemiological modelling also provides a basis for the hybrid extension proposed in this paper. Some compartmental models account for individuals who experience more than one transmission pathway at the same time, such as co-circulating strains or dual-exposure groups. These models typically introduce a separate joint compartment that represents individuals exposed to both pathways, rather than including

them in either single-pathway compartment. The Hybrid compartment introduced in Section 4 follows this same logic, applied to concurrent traditional and AI-assisted engagement.

2.4. APOS Theory and the Definition of Mastery

APOS theory (Dubinsky, 1991; Asiala et al., 1996; Arnon et al., 2014) describes conceptual development in mathematics and related disciplines as progressing through four stages. At the Action stage, a learner can execute a procedure only in response to external cues or step-by-step instructions. At the Process stage, the procedure has been interiorised: the learner can describe, reverse, or reflect on it mentally without performing every step. At the Object stage, the process it is becomes a mental entity that the learner can act upon, for instance comparing, combining, or transforming it. At the Schema stage, actions, processes, and objects relating to a concept are organised into a coherent framework that the learner can invoke flexibly, including in novel or unfamiliar situations.

APOS theory has been used extensively to design and evaluate instruction in undergraduate mathematics, but it has not, to the author's knowledge, been used to operationally define the outcome state of a compartmental learning-evaluation model. The DPLE model's Mastery compartment, by defining achievement only as reaching "curriculum objectives," leaves the assessment criterion for entry into M unspecified and, in practice, at the discretion of individual instructors. Section 4.4 addresses this by using the Schema level of APOS theory as the explicit, assessable criterion for classification into the Mastery compartment of the model developed in this paper.

3. Methodology

3.1. Design Principles of the DHLE Model

The Dynamic Hybrid Learning Evaluation (DHLE) model was developed according to the following principles, extending those underlying the DPLE model:

- The model must retain interaction-driven transitions, in which the influence of existing traditional and AI-assisted learners recruits unengaged students, as in the DPLE model.
- The model must represent students who engage with both traditional instruction and AI-assisted learning concurrently through an explicit hybrid compartment, fed by interaction between the two single-pathway populations rather than by direct recruitment from the unengaged pool.
- The model must consolidate achievement outcomes from traditional, AI-assisted, and hybrid into a single Mastery compartment.
- Entry into the Mastery compartment must correspond to an explicit, assessable pedagogical criterion, defined here via the Schema level of APOS theory, rather than an unspecified notion of curriculum completion.
- The model should be easy to analyse and estimate using classroom data, using the same finite-difference and least-squares methods as the DPLE model.
- The model must include comparative indices, analogous to the DPLE model's Mastery Efficiency Ratio, to quantify the relative efficiency of AI-assisted, traditional, and hybrid learning pathways.

3.2. Analogy with Multi-Pathway Compartmental Models

As with the TPL and DPLE models, the DHLE model draws its structural inspiration from the SIR epidemiological framework, in which susceptible individuals are recruited into an infected compartment through contact with existing infected individuals at a rate proportional to both populations. The DHLE model applies this logic twice over: unengaged students (U) are recruited into traditional learning (T) through interaction with existing traditional learners at rate α , and into AI-assisted learning (A) through interaction with existing AI-assisted learners at rate θ , producing the nonlinear terms αUT and θUA .

The hybrid extension follows the logic used in compartmental models of dual or co-exposure: A student already engaged through one pathway may adopt the other pathway as a supplementary source of support through interaction with its learning environment and may eventually rely equally on both pathways. For instance, a traditional learner (T) who interacts with AI-assisted learners (A) may begin to rely equally on AI-assisted learning and traditional learning, thereby moving into the Hybrid compartment (H) at rate σ , producing the interaction term σTA . Similarly, an AI-assisted learner who interacts with the traditional learning environment may begin to rely equally on traditional instruction and AI-assisted learning, thereby moving into H at rate ρ , producing the term ρTA . Because σ and ρ need not be equal, traditional and AI-assisted learners may adopt the complementary pathway at different rates. Therefore, the two hybridisation terms are retained as structurally identical but numerically distinct interaction effects. Full mathematical detail is provided in Section 4.3.

4. Model Development and Discussion

4.1. The DHLE Model Structure

The DHLE model partitions the student population into five compartments, illustrated in Figure 1:

- Unengaged students (U): students who have not yet been recruited into any learning pathway. On the

first day of the course, all students are assumed to begin in this compartment.

- Traditional learning (T): students who primarily engage with the curriculum through instructor-led and conventional learning activities, such as lectures, textbooks, tutorials, and classroom-based activities, with limited or no reliance on AI-assisted learning support.
- AI-assisted learning (A): students actively engaged with the curriculum primarily through AI-assisted learning, with limited or no reliance on traditional instructor-led instruction.
- Hybrid students (H): students who equally rely on both traditional instruction and AI-assisted learning.
- Mastery (M): students who have been assessed, per the APOS-based criterion defined in Section 4.4, as having reached the Schema level of conceptual understanding of the curriculum content, irrespective of the pathway through which mastery was attained.

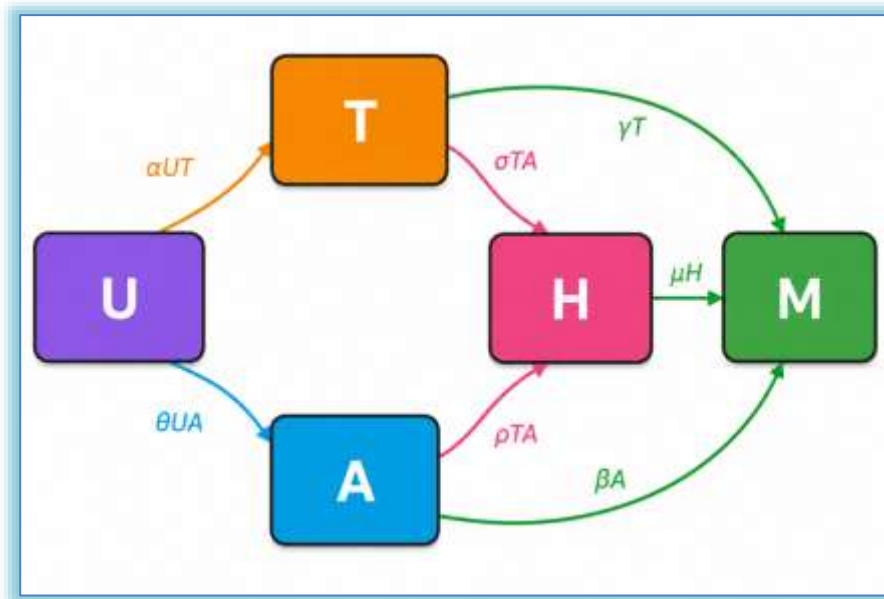


Figure 1. The Dynamic Hybrid Learning Evaluation (DHLE) Model. U = Unengaged; T = Traditional learners; A = AI-assisted learners; H = Hybrid learners; M = Mastery.

4.2. Model Parameters

The DHLE model is governed by seven parameters:

- **Alpha (α):** the rate at which traditional learners (T) recruit unengaged students (U) into traditional learning. Measures how strongly traditional teaching converts unengaged students into active traditional learners.
- **Theta (θ):** the rate at which AI-assisted learners (A) recruit unengaged students (U) into AI-assisted learning. Measures how strongly exposure to AI-assisted learning converts unengaged students into active AI-assisted learners.
- **Sigma (σ):** the rate at which traditional learners (T) adopt AI-assisted learning as a supplementary pathway and eventually rely equally on both pathways, and moved from T into the Hybrid compartment (H).
- **Rho (ρ):** the rate at which AI-assisted learners (A) adopt traditional instruction as a supplementary pathway and eventually rely equally on both pathways, and moved from A into the Hybrid compartment (H).
- **Gamma (γ):** the transition rate at which traditional learners move from active engagement to mastery (M). Reflects the efficiency of traditional instruction, in isolation, in producing Schema-level mastery.
- **Beta (β):** the transition rate at which AI-assisted learners move from active engagement to mastery (M). Reflects the efficiency of AI-assisted learning, in isolation, in producing Schema-level mastery.
- **Mu (μ):** the transition rate at which hybrid learners move from active engagement to mastery (M). Reflects the efficiency of combined traditional and AI-assisted engagement in producing Schema-level mastery, and need not equal γ , β , or any simple combination of the two.

4.3. Mathematical Formulation

The DHLE model is described by the following system of ordinary differential equations (ODEs):

$$\left. \begin{aligned} \frac{dU}{dt} &= -\alpha UT - \theta UA & (a) \\ \frac{dT}{dt} &= \alpha UT - \gamma T - \sigma TA & (b) \\ \frac{dA}{dt} &= \theta UA - \beta A - \rho TA & (c) \\ \frac{dH}{dt} &= \sigma TA + \rho TA - \mu H & (d) \\ \frac{dM}{dt} &= \gamma T + \beta A + \mu H & (e) \end{aligned} \right\} \quad (1)$$

Equation (1a) describes the rate of change of the unengaged compartment. The term $-\alpha UT$ represents unengaged students recruited into traditional learning; the term $-\theta UA$ represents unengaged students recruited into AI-assisted learning. Both terms are negative because they deplete the unengaged pool. The unengaged population decreases as both traditional and AI-assisted learners draw new students into active engagement.

Equation (1b) describes the traditional-learner compartment. The growth term αUT represents recruitment from the unengaged pool. Two loss terms apply: $-\gamma T$ represents traditional learners who progress to mastery, and $-\sigma TA$ represents traditional learners who moved into the Hybrid compartment.

Equation (1c) mirrors (1b) for the AI-assisted compartment: the growth term θUA represents recruitment from the unengaged pool, $-\beta A$ represents AI-assisted learners who progress directly to mastery, and $-\rho TA$ represents AI-assisted learners who moved into the Hybrid compartment.

Equation (1d) describes the Hybrid compartment. It is fed exclusively by the two hybridisation terms, σTA and ρTA , reflecting that hybrid engagement in this model arises from an existing single-pathway learner adopting the complementary pathway, rather than from direct recruitment out of the unengaged pool. The loss term $-\mu H$ represents hybrid learners who progress to mastery.

Equation (1e) accumulates all students who have achieved mastery, from any of the three engaged pathways: γT from traditional learning, βA from AI-assisted learning, and μH from hybrid engagement. The DHLE model rests on the same three underlying conditions as the DPLE model:

4.4. APOS-Defined Mastery Compartment

The DPLE model defines entry into its Mastery compartment only as having “achieved the predetermined curriculum objectives,” without a specified assessment criterion. The DHLE model replaces this with an explicit, assessable criterion grounded in APOS theory (Dubinsky, 1991; Arnon et al., 2014). It is important to note the scope of this use of APOS theory: it defines what counts as membership in the Mastery compartment (M), it is a measurement and classification instrument, applied at the point of assessment, and it does not introduce Action, Process, Object, or Schema as additional compartments or transitions within the ODE system of Section 4.3. A student's trajectory through U, T, A, and H is governed entirely by equations (1a)–(1d); APOS enters only in determining, at each observation point, whether a given student's demonstrated understanding qualifies them to be counted in M. Table 1, summarises the four APOS levels and the classification rule used to determine mastery status.

Table 1. APOS-based classification rubric defining entry into the Mastery compartment (M).

APOS Level	Description of Learner Understanding (Dubinsky, 1991; Arnon et al., 2014)	Classification Rule for M
Action	The learner can execute a taught procedure only when prompted step-by-step, or with external cues (e.g. from an instructor, a worked example, or an AI tutor's guidance).	Not counted in M; classified as still active in T, A, or H.
Process	The procedure has been interiorised: the learner can describe, reverse, or reflect on it mentally, without performing every external step.	Not counted in M; classified as still active in T, A, or H.
Object	The learner treats the process as a mental entity that can be compared, combined, or transformed, but cannot yet integrate it flexibly with other concepts.	Not counted in M; classified as still active in T, A, or H.
Schema	Actions, processes, and objects relating to the concept are organised into a coherent framework the learner can invoke flexibly, including in novel or unfamiliar problem situations.	Counted in M. This is the sole entry criterion into the Mastery compartment.

This definition has a practical consequence for Step 1 of the application procedure (Section 4.7): the assessment instrument used to classify students at each observation point must be capable of distinguishing Schema-level understanding from Object-, Process-, or Action-level performance. For

example, through tasks that require a learner to apply a concept in an unfamiliar context, rather than only to reproduce a taught procedure. Instruments that only test procedural recall are, under this definition, unable to identify true entry into M and will systematically over-classify Action- or Process-level students as having achieved mastery.

4.5. Relative Mastery Efficiency

The Relative Mastery Efficiency (RME) is a normalised index whose purpose is to express each pathway's mastery transition rate as a proportion of the fastest-performing pathway, so that traditional, AI-assisted, and hybrid learning can be ranked directly against one another in a single step, rather than compared only two at a time as under the DPLE model's pairwise ratio. It is defined separately for each of the three pathways, as follows:

Case 1: Evaluation of Traditional learning pathway,

$$RME_T = \frac{\gamma}{\max(\beta, \mu)} \quad (2a)$$

- If $RME_T > 1$, the traditional-learning mastery transition rate is the highest of the three pathways, higher than both the AI-assisted and the hybrid-learning pathways. Hence, traditional learning is therefore the most strongly recommended pathway.
- If $RME_T < 1$, the traditional-learning mastery transition rate is not the highest of the three pathways, at least one of the AI-assisted or hybrid-learning pathways produces mastery faster. Hence, traditional learning is therefore not the most strongly recommended pathway.
- If $RME_T = 1$, the mastery transition rates are equal, meaning that traditional learning students and either the AI-assisted or hybrid-learning pathways achieve mastery at the same rate. Hence, the pathway that is most readily accessible to students at that time is recommended.

Case 2: Evaluation of AI-assisted learning pathway,

$$RME_A = \frac{\beta}{\max(\gamma, \mu)} \quad (2b)$$

- If $RME_A > 1$, the AI-assisted mastery transition rate is the highest of the three pathways, higher than both the traditional and the hybrid-learning pathways. Hence, AI-assisted learning is therefore the most strongly recommended pathway.
- If $RME_A < 1$, the AI-assisted mastery transition rate is not the highest of the three pathways, at least one of the traditional or hybrid-learning pathways produces mastery faster. Hence, AI-assisted learning is therefore not the most strongly recommended pathway.
- If $RME_A = 1$, the mastery transition rates are equal, meaning that AI-assisted learning students and either the traditional or hybrid-learning pathways achieve mastery at the same rate. Hence, the pathway that is most readily accessible to students at that time is recommended.

Case 3: Evaluation of Hybrid learning pathway,

$$RME_H = \frac{\mu}{\max(\gamma, \beta)} \quad (2c)$$

- If $RME_H > 1$, the hybrid mastery transition rate is the highest of the three pathways, higher than both the AI-assisted and traditional-learning pathways. Hence, hybrid learning is therefore the most strongly recommended pathway.
- If $RME_H < 1$, the hybrid mastery transition rate is not the highest of the three pathways, at least one of AI-assisted and traditional-learning pathways produces mastery faster. Hence, hybrid learning is therefore not the most strongly recommended pathway.
- If $RME_H = 1$, the mastery transition rates are equal, meaning that hybrid learning students and either the AI-assisted and traditional-learning pathways achieve mastery at the same rate. Hence, the pathway that is most readily accessible to students at that time is recommended.

4.6. Comparison with the DPLE Model

Table 2, summarises the structural differences between the DPLE model and the DHLE model developed in this paper.

Table 2. Structural comparison of the DPLE model and the DHLE model.

Feature	DPLE Model (Mazibuko, 2026b)	DHLE Model (This Study)
Compartments	Four: U, T, P (peer), M	Five: U, T, A (AI-assisted), H (hybrid), M
Peer identity	Human peer learners	AI-assisted learning (conversational/adaptive AI tutoring)
Hybrid engagement	Not represented; students assumed to occupy exactly one pathway	Explicit Hybrid compartment H, fed by σTA and ρTA interaction terms
Mastery definition	Achievement of unspecified "curriculum objectives"	Achievement of Schema-level understanding per APOS theory (Table 1)
Comparative indices	Mastery Efficiency Ratio (MER); Threshold Value (TV)	Relative Mastery Efficiency (RME)

Parameters	Four: $\alpha, \theta, \gamma, \beta$	Seven: $\alpha, \theta, \sigma, \rho, \gamma, \beta, \mu$
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4.7. Application Procedure

The DHLE model is applied through a seven-step procedure, extending the six-step DPLE procedure with an explicit APOS-based mastery-assessment step.

Step 1 — Collect longitudinal classroom data. At regular intervals (weekly or per lesson), classify each student into one of the five DHLE compartments: U (unengaged), T (traditional only), A (AI-assisted only), H (hybrid), or M (mastered). Classification into M must follow the APOS-based rubric in Table 1, using assessment tasks capable of distinguishing Schema-level understanding from Action-, Process-, or Object-level performance.

Step 2 — Design or select an APOS-aligned mastery instrument. Before or alongside data collection, validate an assessment instrument (e.g. a rubric-scored task set or structured interview protocol) capable of reliably distinguishing the four APOS levels for the specific curriculum content, so that classification into M in Step 1 is defensible.

Step 3 — Estimate observed rates of change. For each compartment $X \in \{U, T, A, H, M\}$ and time interval Δt , approximate the derivative using finite differences:

$$\frac{dX}{dt} = \frac{X_{t+1} - X_t}{\Delta t} \quad (3)$$

Step 4 — Solve for parameters using the model equations. Substitute the observed compartment values (Step 1) and observed derivatives (Step 3) into equations (1a)–(1e) at each time point, generating an over-determined system of equations in the seven unknown parameters $\alpha, \theta, \sigma, \rho, \gamma, \mu$, and β .

Step 5 — Estimate parameters using regression optimisation. Estimate the seven parameters by minimising the total squared error between observed and model-predicted compartment values:

$$\text{Minimize } \Sigma (\text{Observed} - \text{Predicted})^2 \quad (4)$$

Observed: the derivatives computed from your real classroom data in Step 3

Predicted: what the model's equations (1a – 1e) say those derivatives should be when substituting the parameters ($\alpha, \theta, \sigma, \rho, \gamma, \mu$, and β) obtained in step 4.

Step 6 — Validate the DHLE model. Using the fitted parameters, generate predicted compartment and compare them with the observed data. If discrepancies are statistically insignificant (using an error measure at the researcher's discretion), the model may be regarded as valid for the dataset under consideration.

Observed: the same raw compartment counts from step 1, the actual number (or proportion) of students in U, T, A, H, and M that you recorded at each observation time point. Same source as always; nothing new is collected.

Predicted: generated by numerically solving (integrating) the ODE system (1a)–(1e) forward in time, starting from the initial conditions ($U(0) = N, T(0) = A(0) = H(0) = M(0) = 0$), using the parameter values you just fitted in Step 5. This produces a full simulated trajectory predicted $U(t), T(t), A(t), H(t), M(t)$ curves across the whole observation period which you then read off at the same time points as your observed data.

Step 7 — Compute Relative Mastery Efficiency (RME). Using the parameters fitted in Step 5, compute RME_T, RME_A , and RME_H . As defined in Section 4.5, use these values to identify the fastest-performing pathway and rank traditional, AI-assisted, and hybrid learning accordingly. That will complete the purpose of the model.

5. Conclusions

This paper has presented the Dynamic Hybrid Learning Evaluation (DHLE) model as a structural extension of the Dynamic Peer Learning Evaluation (DPLE) model (Mazibuko, 2026b), it is an extension of the original Traditional versus Peer Learning (TPL) model (Mazibuko, 2026). Three extensions were introduced. First, the peer-learning pathway was redefined as an AI-assisted learning pathway, reflecting the increasing role of conversational and adaptive AI tutoring systems in providing dialogic, on-demand instructional support. Second, an explicit Hybrid compartment was introduced to represent students who draw on both traditional and AI-assisted learning concurrently, together with interaction-driven hybridisation terms (σ_{TA} and ρ_{TA}) governing entry into this compartment. Third, the Mastery compartment was re-specified using APOS theory, so that entry into it corresponds to an explicit, assessable criterion, attainment of the Schema level of conceptual understanding, rather than an unspecified notion of curriculum completion; APOS was applied strictly as a classification instrument for the Mastery state and was not introduced as an additional set of ODE transitions.

A Relative Mastery Efficiency (RME) was derived from the resulting five-compartment, nonlinear ODE system. It provides a normalised measure for ranking the traditional, AI-assisted, and hybrid learning pathways according to their mastery transition rates, with each pathway expressed relative to the fastest-

performing pathway. Unlike the DPLE model, which relies on pairwise comparisons, the RME enables all three pathways to be compared simultaneously. Furthermore, a Seven-step application procedure was provided, extending the DPLE model's six-step procedure with an explicit step for designing or validating an APOS-aligned mastery-assessment instrument.

As with the TPL and DPLE models, the DHLE model has not yet been empirically validated against classroom data. This is a deliberate choice, not an oversight: the model's structure, parameters, and estimation method needed to be fully developed and checked before being tested on real data. Genuine validation also requires a course with a

real mix of traditional, AI-assisted, and hybrid learners, and a mastery test that can reliably distinguish APOS levels (Section 4.4). Publishing the model first allows other researchers to scrutinise it and apply it in their own classrooms, rather than relying on results from a single study. The model is now sufficiently developed for empirical validation to begin with confidence. The next phase of this research will apply it to longitudinal data from courses that combine traditional instruction with an AI tutoring system, using APOS-aligned assessment instruments to classify students into the Mastery compartment, in order to evaluate the model's goodness-of-fit, refine the parameter-estimation procedure, and test the practical value of the Relative Mastery Efficiency.

STATEMENTS AND DECLARATIONS

The author gratefully acknowledges the support of the New Generation of Academics Programme (nGAP) for providing financial assistance towards the publication fees of this article. Their support is sincerely appreciated and contributed significantly to the successful dissemination of this research.

In the current study, AI (ChatGPT, Copilot and Claude) assisted tools were used for grammar correction purposes only. All mathematical content, model development, and intellectual contributions are solely those of the author.

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