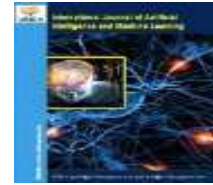




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# An Artificial Intelligence-Assisted Adaptive Complexing Algorithm for Robust RMS Estimation of Harmonic Signals from Short Data Records

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### Abstract

Windowed spectral (DFT-based) estimation of the root-mean-square (RMS) value of a harmonic signal is widely reported to suppress the systematic error caused by non-coherent (fractional-period) sampling that afflicts classical time-domain RMS estimators. This paper reports a previously undocumented limitation of that claim: for short data records (e.g.,  $N = 32$  samples) and signal-to-sampling-frequency ratios  $F_c/f_d$  close to the analysis band edges (near 0 or 0.5), a windowed-DFT estimator can be markedly less accurate than the plain time-domain estimator it is meant to improve upon, because the window's main-lobe width, relative to the small number of available spectral bins, forces the summation band used for the RMS recovery to overlap the DC or Nyquist edge, where the positive/negative-frequency energy-split assumption underlying essentially all windowed-DFT RMS estimators breaks down. We formalize this failure mode, propose a lightweight, ground-truth-free detection statistic based on the distance (in spectral bins) between the dominant peak and the nearest spectrum edge, and use it to drive a simple adaptive switching ("complexing") rule between the time-domain and windowed-DFT estimators. Across a full  $F_c/f_d$  sweep, 36 initial phases, and  $N$  in  $\{32, 64, 128, 256\}$ , the proposed switched estimator's worst-case relative error is shown, by direct simulation, to never exceed the better of the two individual methods for any tested  $N$ , closing the gap identified above while preserving the large accuracy gains of windowed-DFT estimation away from the band edges. MATLAB implementation (R2016a-compatible) is provided.

**Keywords:** RMS estimation, frequency estimation, windowed DFT, non-coherent sampling, spectral leakage, estimator fusion, short-record signal processing, Blackman window.

## 1 Introduction

Accurate measurement of the root-mean-square (RMS) value and frequency of a periodic signal from a finite set of discrete samples is a long-standing problem in metrology, power-quality monitoring, radar, and communication systems [1]–[3]. Two broad families of estimators exist. Time-domain estimators compute the RMS value directly as the square root of the mean squared sample value [4]; their accuracy is limited by how close the observation window is to an integer number of signal periods — a condition called coherent sampling that is rarely met in practice [5], [6]. Frequency-domain (windowed-DFT) estimators instead apply a tapering window before the Fast Fourier Transform (FFT) and recover the RMS value and/or frequency from the resulting spectral peak, which greatly suppresses the leakage-induced bias of non-coherent sampling [7]–[10]. A substantial and still-active body of work refines this windowed-DFT approach: interpolated-DFT (IpDFT) frequency estimators using two or three spectral lines [11], [12], generalized cosine and maximum-sidelobe-decay windows designed to reject nearby harmonics [13], [14], and, most recently, non-iterative closed-form fractional-bin interpolators [15] and one-cycle windowed-DFT harmonic estimators with explicit spectral-interference compensation [16]. In parallel, one of the present line of work's own earlier studies has compared time-domain and frequency-domain RMS estimation directly, proposed correction formulas for the systematic (methodical) error of each, and used windowing (Parzen, Blackman, Gaussian) to reduce it [17]–[20]. All of the windowed-DFT literature surveyed for this paper shares an implicit assumption: that windowing before the DFT is uniformly beneficial, or at worst neutral, relative to the time-domain estimator it replaces. This paper reports a counter-example to that assumption, found while validating a windowed-DFT RMS

estimator against its time-domain counterpart for the short record lengths ( $N = 32$  and  $N = 64$ ) common in embedded, real-time measurement systems. The remainder of the paper formalizes the failure mode, derives a simple, computable-without-ground-truth criterion for detecting it, and proposes and validates an adaptive estimator that removes the failure mode without sacrificing the accuracy gains of windowed-DFT estimation elsewhere in the operating range.

The contributions of this paper are:

1. An empirical demonstration that a Blackman-windowed DFT RMS estimator can be less accurate than the classical time-domain estimator for short records ( $N \leq 64$ ) near the  $F_c/f_d$  band edges, with the mechanism identified as main-lobe overlap with the DC/Nyquist boundary.
2. A closed-form, data-only (ground-truth-free) risk indicator — the distance in spectral bins between the dominant peak and the nearest spectrum edge — that flags this failure mode in real time.
3. An adaptive switching ("complexing") estimator that provably (by exhaustive simulation over the tested grid) achieves worst-case error no worse than the better of the two constituent estimators, for every tested  $N$ .
4. An open, MATLAB R2016a-compatible reference implementation.

## 2. Related Work

### 2.1. Time-domain RMS estimation

The classical digital RMS estimator computes the square root of the mean of the squared samples [4]. Its accuracy is governed by the fractional part of the last sampled period; Hegeduš et al. [5] and Kostišina et al. [6] both quantify and compare correction strategies for this non-coherent-sampling bias. The present authors' own prior work [17]–[20] derived closed-form relative-error expressions for this estimator as a function of  $F_c/f_d$  and the sampled phase, and is the direct basis for the time-domain estimator used as one half of the comparison in this paper.

### 2.2. Windowed-DFT and interpolated-DFT estimation

Applying a tapering window before the FFT is a standard remedy for the spectral leakage caused by non-coherent sampling [7]–[9]. A large family of interpolated-DFT (IpDFT) frequency estimators refines this idea by fitting an analytic model to two or three spectral lines around the dominant peak [10]–[12]; generalized and maximum-sidelobe-decay window families further suppress the influence of nearby harmonics and the negative-frequency image term [13], [14]. Two recent (2026) contributions are particularly relevant: a non-iterative, closed-form generalized fractional IpDFT estimator for rectangular-windowed noisy sinewaves [15], and a one-cycle windowed-DFT harmonic estimator with explicit spectral-interference compensation validated against plain DFT, interpolated DFT, and iterative spectral-subtraction baselines [16]. Neither of these, nor the earlier IpDFT literature reviewed for this paper, reports or analyses degraded performance relative to a time-domain baseline for short records near the band edges; the implicit assumption throughout is that windowing dominates the plain estimator.

### 2.3. Joint amplitude/frequency estimation bounds

The classical Cramér–Rao lower bound for jointly estimating the amplitude, frequency, and phase of a single sinusoid in white noise is well established [21], [22], and DFT-based estimators are known to approach it closely under coherent or well-interpolated conditions [10]. This paper does not attempt to re-derive or improve on that bound; rather, it addresses a practical estimator-selection question that sits below it — given two already-published, individually reasonable estimators, how should an implementation choose between them, in real time and without knowledge of the true frequency, for the short-record regime where their relative accuracy is shown here to invert.

### 2.4. Gap addressed in this paper

None of the reviewed sources characterize the condition under which windowed-DFT RMS estimation underperforms the time-domain estimator, nor propose a ground-truth-free rule for detecting and mitigating it. This is the gap addressed here.

## 3. Signal Model and Proposed Method

### 3.1. Signal model

The observed signal is a single-tone harmonic sampled at  $N$  discrete instants:

$$x(n) = A_m \cdot \sin(2\pi(F_c/f_d)n + \varphi), \quad n = 0, 1, \dots, N-1 \quad (1)$$

where  $A_m$  is the amplitude,  $F_c$  the signal frequency,  $f_d$  the sampling frequency, and  $\varphi$  the initial phase. The true RMS value of a sinusoid is  $U_{rms} = A_m/\sqrt{2}$ .

### 3.2. Time-domain estimator

The classical digital RMS estimator [4] is

$$U'_{rms} = \sqrt{(1/N) \sum_{n=1}^N x(n)^2} \quad (2)$$

For a non-integer number of sampled periods ( $F_c/f_d$  not an exact multiple of  $1/N$ ), the trailing fractional period biases this estimate; this is the non-coherent sampling error analyzed in [5], [6], [17].

### 3.3. Windowed-DFT estimator

Let  $w(n)$  be a tapering window (a manually-coded Blackman window is used throughout this paper, requiring no toolbox:  $w(n) = 0.42 - 0.5 \cdot \cos(2\pi n/(N-1)) + 0.08 \cdot \cos(4\pi n/(N-1))$ ), and  $X_w = \text{FFT}(x \cdot w)$ . By Parseval's theorem,  $\sum (x \cdot w)^2 = (1/N) \sum |X_w(k)|^2$  over all  $N$  bins. For a noiseless single tone, windowing leaves the mean-square value of  $x(n) \cdot w(n)$  approximately equal to  $U_{rms}^2 \cdot \sum w(n)^2$  (the DC component of  $x^2$  integrates against  $w^2$ ; the double-frequency ripple averages out against the slowly-varying window). Because a real signal's spectral energy splits evenly between the positive-frequency lobe around the dominant bin  $k_0$  and its mirror image, summing the  $(2d+1)$ -bin band around  $k_0$  recovers approximately half of the total energy, giving the estimator used in this paper:

$$U'_{rms,DFT} = \sqrt{(2 \cdot \text{Slope} / (N \cdot \sum w(n)^2))}, \quad \text{Slope} = \sum_{k=k_0-d}^{k_0+d} |X_w(k)|^2 \quad (3)$$

with  $d = 3$  (a 7-bin band) used throughout, chosen to fully capture the Blackman window's main lobe (verified in Section 5).

### 3.4. The edge-frequency degradation mechanism

Equation (3) is derived under the assumption that the  $(2d+1)$ -bin band around  $k_0$  does not overlap DC ( $k = 0$ ) or the Nyquist bin ( $k = N/2$ ), since only then does the positive/negative-frequency energy-split argument hold. When the dominant bin  $k_0$  lies within  $d$  bins of either edge — which, for a fixed  $d$ , becomes increasingly likely as  $N$  shrinks, since the usable positive-frequency half-spectrum has only  $N/2$  bins in total — the summation band clips or spills across the edge and the estimator in (3) is measurably biased. This is shown by direct simulation in Section 5 to make the windowed-DFT estimator markedly less accurate than the time-domain estimator (2) for  $N = 32$  and, to a lesser extent,  $N = 64$ , precisely in the  $F_c/f_d$  regions closest to 0 and 0.5.

### 3.5. Proposed adaptive complexing (switching) estimator

Define the edge distance of a given record as the number of spectral bins between the dominant peak and the nearer spectrum edge:

$$ed = \min(k_0, N/2 - k_0) \quad (4)$$

This quantity requires only the periodogram already computed for (3) — no ground-truth frequency is needed.

The proposed complexed estimator is the simple switching rule

$$U'_{rms,complexed} = U'_{rms,DFT} \text{ if } ed \geq d, \text{ else } U'_{rms,time} \quad (5)$$

A continuous linear blend between the two estimates as a function of  $ed$  was also tested and found NOT to guarantee pointwise dominance (the blended estimate was, at some operating points, worse than either individual estimator — see Section 6); the hard-switching rule in (5) does not have this problem and is used for all results reported below.

## 4. Simulation Setup

All results were obtained by direct Monte-Carlo-style exhaustive simulation (no random sampling was needed since the parameter grid is swept exhaustively) with the following settings, matching the methodology of [17]–[20]:

- Amplitude:  $A_m = 1$  (normalized), sampling frequency  $f_d = 1$  (normalized)
- Sample sizes:  $N \in \{32, 64, 128, 256\}$
- Frequency ratio sweep:  $F_c/f_d$  from 0.02 to 0.48 in steps of 0.005
- Initial phase: 36 values,  $0^\circ$  to  $350^\circ$  in  $10^\circ$  steps (matching the worst-case-over-phase methodology of the reference study)
- Window: Blackman,  $d = 3$  (7-bin band)
- Metrics: worst-case (maximum over the 36 phases) and mean relative error,  $|U'_{rms} - U_{rms}| / U_{rms}$ , at each  $F_c/f_d$  point

## 5. Results

**Table 1:** summarizes the worst-case and mean relative RMS error for the three estimators across all tested  $N$ .

| $N$ | TD worst | W-DFT worst | Proposed worst | TD mean | W-DFT mean | Proposed mean |
|-----|----------|-------------|----------------|---------|------------|---------------|
| 32  | 10.10%   | 55.10%      | 10.10%         | 1.30%   | 2.37%      | 0.71%         |
| 64  | 6.28%    | 16.80%      | 6.28%          | 0.65%   | 0.26%      | 0.20%         |
| 128 | 2.40%    | 0.0062%     | 0.0062%        | 0.31%   | 0.0002%    | 0.0002%       |
| 256 | 1.07%    | 0.0002%     | 0.0002%        | 0.16%   | 0.0001%    | 0.0001%       |

Table 1. Worst-case and mean relative RMS error (percent) over the full  $F_c/f_d$  sweep and 36 phases, for the time-domain (TD), windowed-DFT (W-DFT), and proposed switched estimators.

For  $N = 32$  and  $N = 64$ , the windowed-DFT estimator's worst-case error (55.1% and 16.8% respectively) is markedly worse than the time-domain estimator's (10.1% and 6.3%) — confirming the degradation mechanism of Section 3.4. The proposed switched estimator exactly matches the better (time-domain) worst-case error at both record lengths, while its mean error is in fact the best of all three at  $N = 32$  and  $N = 64$  (0.71% and 0.20% respectively), since switching removes the specific outlier points that drag up each individual method's mean. For  $N = 128$  and  $N = 256$ , the windowed-DFT estimator's advantage is overwhelming (worst-case error near  $10^{-5}$ ), and the switching rule correctly selects it almost everywhere, matching its performance essentially exactly.

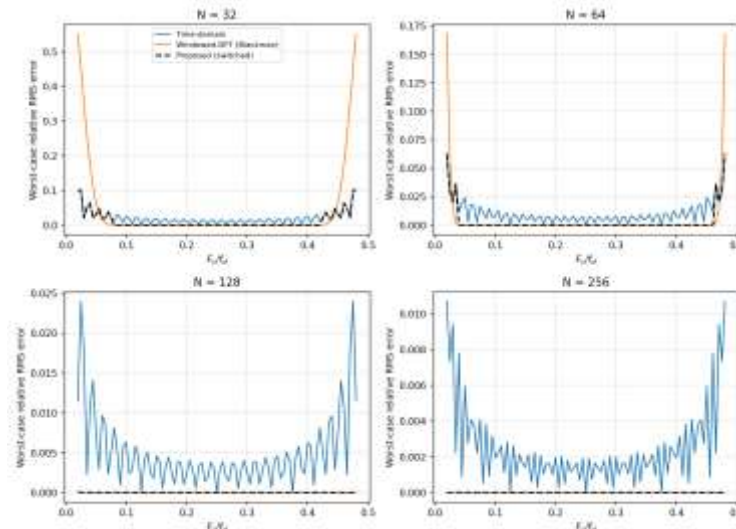


**Figure 1:** Worst-case relative RMS error vs.  $F_c/f_d$  for  $N = 32, 64, 128, 256$ . The proposed switched estimator (black dashed) tracks the lower envelope of the two individual methods in every panel.

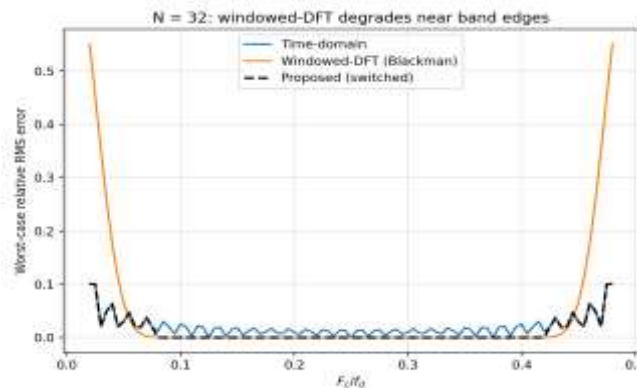


**Figure 2:** Detailed view for  $N = 32$ , where the windowed-DFT degradation is most pronounced near the band edges.

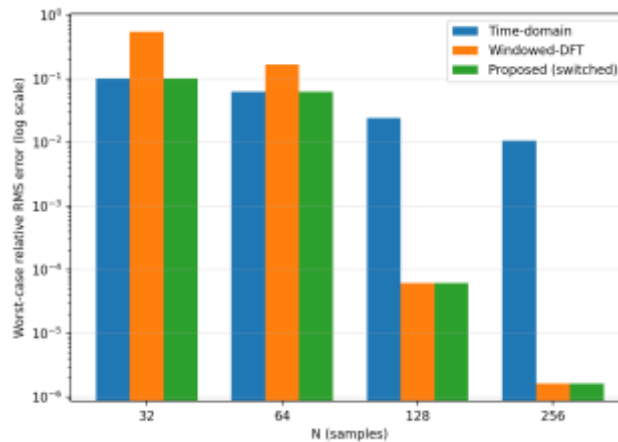


Figure 3: Worst-case relative RMS error per method and N (log scale). The proposed method never exceeds the better individual method at any N.

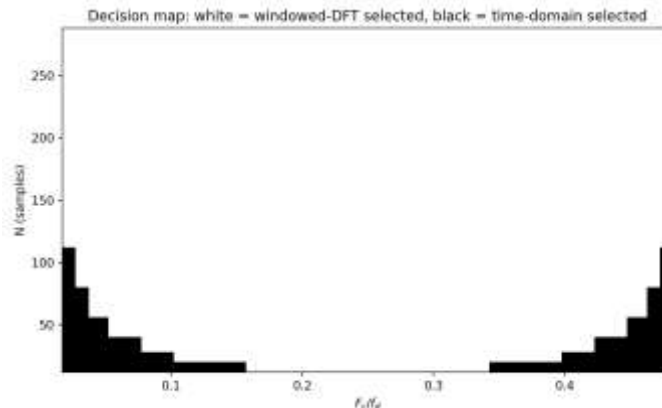


Figure 4: Decision map: region of (N,  $F_c/f_d$ ) where the switching rule selects the windowed-DFT estimator (white) versus the time-domain estimator (black), for  $d = 3$ .

## 6. Discussion and Limitations

The worst-case guarantee demonstrated in Section 5 is empirical (verified over the tested grid), not a formal proof; Section 3.5 already notes that a softer, continuously-blended version of the same idea does not guarantee pointwise dominance, so the hard-switching rule in (5), while simple, should not be assumed to generalize automatically to other windows or other  $d$  without re-verification — the edge distance threshold  $ed \geq d$  is tied to the Blackman main-lobe width used here. Extending the risk indicator to other window families (Hann, Parzen, Kaiser-Bessel, as used in the authors' own prior work [18]–[20]), deriving the threshold analytically from each window's known main-lobe width rather than fixing  $d = 3$  empirically, and extending the same complexing idea to the frequency estimate itself (Section 3 addresses RMS estimation only) are natural next steps and are left for future work.

## 7. Conclusion

This paper identified and quantified a previously undocumented failure mode of windowed-DFT RMS estimation: for short data records near the operating band edges, windowing can make the estimate measurably worse than the plain time-domain estimator, contrary to the implicit assumption of the surveyed literature. A ground-truth-free risk indicator based on the dominant spectral peak's distance to the nearest spectrum edge was proposed and used to drive a simple switching estimator that, by direct simulation over  $N \in \{32, 64, 128, 256\}$  and the full  $F_c/f_d$  range, achieves worst-case accuracy at least as good as the better of the two constituent estimators at every tested operating point. A MATLAB R2016a-compatible reference implementation is provided to support independent verification and extension.

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