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An Intelligent Hesitation-Weighted Fuzzy Framework For Multi-Objective Reliability Optimization

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Abstract

Reliability optimization of engineering systems often involves multiple conflicting objectives and uncertain parameter information, making classical deterministic optimization approaches inadequate. To address this challenge, this study proposes a novel Hesitation-Weighted Sigmoid Membership Function (HWSMF) within an intuitionistic fuzzy optimization framework for solving multi-objective reliability optimization problems under uncertainty. The proposed membership structure explicitly incorporates the hesitation degree of intuitionistic fuzzy sets, enabling adaptive adjustment of satisfaction and rejection levels based on the degree of uncertainty present in system parameters. Component reliabilities are represented using triangular interval numbers, and the resulting interval-valued objectives are transformed into a deterministic intuitionistic fuzzy programming model. The optimization problem is solved using Particle Swarm Optimization (PSO), and the obtained results are compared with those produced by a Genetic Algorithm (GA) to evaluate solution quality and convergence performance. Numerical experiments on benchmark reliability systems, including series systems and space capsule life-support systems, demonstrate that the proposed HWSMF framework provides improved compromise solutions between system reliability and cost. The results indicate that the PSO-based implementation consistently achieves better convergence and higher solution quality than GA. Overall, the proposed approach offers a flexible and robust decision-making framework for multi-objective reliability optimization in uncertain environments, making it suitable for practical engineering system design problems.

Keyword: Hesitation-Weighted Sigmoid Membership Function, Intuitionistic Fuzzy Sets (IFS), Multi-objective Reliability Optimization, Fuzzy Optimization, Hesitation degree, Pareto Optimal Solutions, Particle Swarm Optimization, Genetic Algorithm.

1. Introduction

Reliability optimization is an important issue in the system design that has been extensively studied over many years. Reliability optimization problem is a nonlinear integer-programming problem of a combinatorial nature that is NP-hard. Researchers have been using both heuristic and enumerative ways to solve this problem [1]. Conventionally, single objective models have been used to solve reliability optimization problems where the component reliability is assumed to be known at specific positive levels between zero and one. However, in actual circumstances, such an assumption is uncommon. Numerous issues, including poor storage facilities, human factors, and environmental factors, affect how reliable components are. Since the likelihood of component reliability is typically unknown in the early stages of design, it is preferable to account for reliability as an inaccurate number. As it can only handle quantitative data, the conventional theory of reliability, which has been based on probabilistic and binary assumptions of state, typically fails to provide practitioners with useful information. In order to address these challenges, the fuzzy set [2] theory-based approaches have been used more frequently in the reliability analysis to propagate uncertainty in the basic events.

Another important extension of the fuzzy set theory is the introduction of intuitionistic fuzzy sets (IFSs) by Atanassov [3-4] that are defined by membership and non-membership functions in contrast with the traditional fuzzy sets that only provide an element with a degree of membership. This extension makes more flexibility in modeling uncertainty, possibly due to the fact that non-membership in a set is not necessarily the opposition of membership. Though intuitionistic fuzzy set (IFS) theory has been applied in solving reliability optimization problems by most researchers, majority of the methods have not exploited the degree of hesitation, which is a significant characteristic of IFSs. As an example, the conventional linear and exponential membership functions, such as the ones employed by Garg et al. [5], assume acceptance or rejection without modifying it in accordance with the hesitation margin which is the measure of uncertainty.

In order to address this problem, we introduce a Hesitation-Weighted Sigmoid Membership Function (HWSMF) that models hesitation adaptively and can give a gradual transition between acceptance and rejection. Sigmoid structure brings about flexibility and the weighting of hesitation directly reflects the uncertainty in the reliability parameters. The proposed algorithm is used to solve multi-objective optimization problems of reliability problems, the uncertain parameters are denoted by triangular intervals. Particle Swarm Optimization (PSO) and benchmarking of a Genetic Algorithm (GA) are used to arrive at the solutions. The principal contributions of the research are the Development of a new HWSMF for intuitionistic fuzzy optimization, the formulation of a multi-objective reliability model in case of parameter uncertainty and the comparative evaluation of PSO and GA to prove the efficiency of the method.

2. Methods and Materials

This section describes the mathematical background, problem formulation, and solution methodology adopted for solving multi-objective reliability optimization problems under uncertainty using intuitionistic fuzzy theory.

2.1 Preliminaries

Intuitionistic Fuzzy Sets

Atanassov [3,4] extended classical fuzzy set theory by introducing intuitionistic fuzzy sets, which incorporate both membership and non-membership functions to better represent uncertainty. This enhancement enables a more flexible and detailed description of imprecise information. Subsequently, Gau and Buehrer [6] proposed the concept of vague sets, which were later demonstrated by Bustince and Burillo [7] to be mathematically equivalent to intuitionistic fuzzy sets.

Definition 1: Let X be a nonempty finite universe. An intuitionistic fuzzy set (IFS) on X is characterized by a membership function $\mu_A(x)$ and a non-membership function $\nu_A(x)$, where

$$\mu_A(x), \nu_A(x) \in [0,1], \qquad 0 \le \mu_A(x) + \nu_A(x) \le 1, \quad \forall x \in X \tag{1}$$

The hesitation degree, representing the uncertainty associated with the element x, is defined as

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \tag{2}$$

When the hesitation degree is zero, the intuitionistic fuzzy set reduces to a classical fuzzy set. The presence of hesitation enables more flexible modeling of uncertainty, which is particularly useful in reliability optimization problems with imprecise data.

Interval Numbers

Definition 2: An interval number A is defined as a closed interval denoted by $A = [a^L, a^R]$, where $a^L, a^R \in \mathbb{R}$ and $a^L \le a^R$.

Alternatively, an interval can be expressed in terms of its midpoint as $A = \langle a^c, a^w \rangle$,

Where $a^c = \frac{a^L + a^R}{2}$ and $a^w = \frac{a^R - a^L}{2}$

Interval representation of reliability parameters

Component reliabilities are rarely known with certainty and are therefore modelled as triangular interval numbers:

$$\tilde{r} = (r^L, r^M, r^U)$$

Where r^L, r^M, and r^U denote the pessimistic, most likely, and optimistic estimates, respectively. This representation allows uncertainty to be incorporated directly into system reliability modelling.

Optimization in Interval Environment

A multi-objective optimization problem involving interval-valued objective functions can be expressed as

Minimize $f(x) = \{f_1(x), f_2(x), \dots, f_b(x)\}$, subject to $x \in S$, Where each objective function $f_t(x)$ is represented by an interval of the form

$$f_t(x) = [f_t^L(x), f_t^C(x), f_t^R(x)], t = 1,2, \dots, b. \quad (3)$$

The feasible set is defined as $S = \{x \mid g_j(x) \leq b_j, j = 1,2, \dots, M\}$.

Pareto Optimal Solution

Definition 3: A feasible solution $x^* \in S$ is considered Pareto optimal if there exists no other feasible solution $x \in S$ such that

$$f_t(x) \leq f_t(x^*) \text{ for all } t,$$

and

$$f_s(x) < f_s(x^*) \text{ for at least one } s.$$

The definitions and formulations presented in this section form the mathematical basis for constructing the proposed hesitation-weighted sigmoid membership function and the subsequent intuitionistic fuzzy optimization model used to address reliability optimization problems under uncertainty.

2.2 Problem Formulation: Reliability Optimization Model

Reliability optimization has been extensively investigated in the literature, and several solution methodologies have been proposed for different system configurations. Early formulations and solution techniques were introduced by Tillman et al. [8], while heuristic-based approaches for complex reliability systems were later explored by Misra and Sharma [9]. Subsequently, intuitionistic fuzzy multi-objective programming methods were applied to reliability optimization by Mahapatra et al. [10,11], and intuitionistic fuzzy techniques were further extended to reliability–redundancy allocation problems by Garg and Sharma [12,13].

To account for data imprecision and incomplete reliability data in practical applications, this section formulates a reliability optimization model under uncertainty, where component reliabilities are represented using triangular interval numbers.

Consider a system consisting of n components, where the reliability of the i^{th} component is denoted by r_i, for $i = 1,2, \dots, n$. The overall system reliability R_s depends on the structural configuration of the components.

For a series system, the system reliability is given by

$$R_s = \prod_{i=1}^{n} r_i \quad (4)$$

For a parallel system, the system reliability is expressed as

$$R_s = 1 - \prod_{i=1}^{n} (1 - r_i) \quad (5)$$

In addition to reliability, the total system cost is an important design consideration and is defined as

$$C_s = \sum_{i=1}^{n} C_i (r_i), \quad (6)$$

where $C_i(r_i)$ represents the cost function with reliability r_i for component i.

The reliability optimization problem aims to determine appropriate component reliability levels that simultaneously maximize system reliability and minimize system cost. This leads to the following bi-objective optimization model:

Maximize: $R_s(r_1, r_2, \dots, r_n)$

Minimize: $C_s(r_1, r_2, \dots, r_n)$

Subject to the constraints:

$$r_{i,min} \leq r_i \leq r_{i,max}, R_{s,min} \leq R_s \leq 1 \text{ for } i = 1,2, \dots, n \quad (7)$$

In real-world system design, component reliabilities are rarely known with certainty. To account for this imprecision, each component's reliability is modeled as a triangular interval number of the form

$$r_i = [a_i, b_i, c_i]$$

Where, $a_i = r_i - I$, $b_i = r_i$, and $c_i = r_i + I$, where I is the predefined uncertainty parameter.

Accordingly, the original problem is transformed into a multi-objective interval optimization problem defined as:

Minimize $\{f_t^L(r), f_t^C(r), f_t^R(r)\}, t = 1,2$

Subject to: $g_j(r) \leq b_j$

$$r_{i,min} \leq r_i \leq r_{i,max}, i = 1,2,\dots,n, r_i \in [0,1] \subset \mathbb{R}$$
$$R_{s,min} \leq R_s \leq 1 \tag{8}$$

This interval-based formulation enables a more realistic representation of uncertainty in reliability parameters. In the following section, the proposed Hesitation-Weighted Sigmoid Membership Function (HWSMF) is introduced to effectively incorporate interval uncertainty within an intuitionistic fuzzy optimization framework.

2.3 Proposed Hesitation-Weighted Sigmoid Membership Function

This section introduces a new Hesitation-Weighted Sigmoid Membership Function (HWSMF) within the intuitionistic fuzzy optimization framework and highlights its advantages over conventional membership formulations, particularly in capturing hesitation and uncertainty more effectively.

Conventional Membership Functions

In intuitionistic fuzzy optimization, membership and non-membership functions are defined separately for each objective function. Most commonly, these functions are expressed in a linear form, where the degree of satisfaction or rejection changes proportionally with the objective value. Due to their simplicity, linear membership functions are widely used in practice.

The conventional linear membership and non-membership functions for an objective function $f_i(x)$are defined as follows:

$$\mu_{f_i}(x) = \begin{cases} 1, & f_i(x) \leq m_i \\ \dfrac{M_i - f_i(x)}{M_i - m_i}, & m_i < f_i(x) < M_i \\ 0, & f_i(x) \geq M_i \end{cases} \tag{9}$$

$$\nu_{f_i}(x) = \begin{cases} 0, & f_i(x) \leq m_i \\ \dfrac{f_i(x) - m_i}{M_i - m_i}, & m_i < f_i(x) < M_i \\ 1, & f_i(x) \geq M_i \end{cases} \tag{10}$$

Where m_i and M_i are respectively the minimum and maximum feasible values of the objective function f_i.

To model nonlinear variations in satisfaction and rejection, exponential-type membership functions have also been proposed. These functions allow greater flexibility than linear forms and are given by:

$$\mu_{f_i}(x) = \begin{cases} 1, & f_i(x) \leq m_i \\ e^{-\alpha\frac{f_i(x)-m_i}{M_i-m_i}} \cdot \frac{e^{\alpha}-1}{e^{\alpha}}, & m_i < f_i(x) < M_i \\ 0, & f_i(x) \geq M_i \end{cases} \tag{11}$$

$$\nu_{f_i}(x) = \begin{cases} 0, & f_i(x) \leq m_i \\ \left(\dfrac{f_i(x) - m_i}{M_i - m_i}\right)^2, & m_i < f_i(x) < M_i \\ 1, & f_i(x) \geq M_i \end{cases} \tag{12}$$

Although these membership structures are capable of representing satisfaction and rejection levels, they do not explicitly incorporate the hesitation margin, which is a fundamental characteristic of intuitionistic fuzzy sets. As a result, their ability to adapt to varying degrees of uncertainty is limited, particularly in highly uncertain decision environments.

To overcome the limitations of conventional membership functions, this study proposes a Hesitation-Weighted Sigmoid Membership Function (HWSMF) that dynamically integrates the hesitation degree into the membership formulation. Unlike traditional approaches that treat all uncertainties uniformly, the proposed function adjusts its behaviour according to the extent of uncertainty associated with each objective.

The key features of the proposed HWSMF are summarized as follows:

- The shape of the membership curve adapts according to the hesitation margin, providing a more realistic representation of uncertainty.
- A sigmoid structure ensures smooth and continuous transitions between acceptance and rejection regions.
- The hesitation degree acts as a weighting factor, enabling the model to respond to imprecise and interval-based data.

The proposed membership function for the objective $f_i(x)$ is defined as:

$$\mu_{f_i}(x) = \begin{cases} 1, & \text{if } f_i(x) \le m_i \\ S(x) \times (1 - \pi(x)), & \text{if } m_i < f_i(x) < M_i \\ 0, & \text{if } f_i(x) \ge M_i \end{cases} \tag{13}$$

where the sigmoid function $S(x)$ is given by

$$S(x) = \frac{1}{1 + e^{-\alpha \frac{f_i(x) - m_i}{M_i - m_i}}} \tag{14}$$

$\pi(x)$ denotes the hesitation degree associated with the objective $f_i(x)$, and $\alpha > 0$ is a control parameter that determines the steepness of the sigmoid curve.
The corresponding non-membership function is defined as:

$$\nu_{f_i}(x) = \begin{cases} 0, & \text{if } f_i(x) \le m_i \\ (1 - S(x)) \times (1 - \pi(x)), & \text{if } m_i < f_i(x) < M_i \\ 1, & \text{if } f_i(x) \ge M_i \end{cases} \tag{15}$$

This adaptive formulation allows the membership and non-membership values to evolve simultaneously with changes in uncertainty, thereby improving the quality of intuitionistic fuzzy decision-making.
The hesitation degree $\pi(x)$is computed based on the relative width of the interval-valued objective function as follows:

$$\pi(x) = \frac{f_i^R(x) - f_i^L(x)}{f_i^{max} - f_i^{min}} \tag{16}$$

where f_i^{max} and f_i^{min} denote the global maximum and minimum values of the objective function over the feasible region. This formulation assigns higher hesitation values to objectives with wider intervals, reflecting greater uncertainty. Consequently, both membership and non-membership measures adjust accordingly.
Overall, the proposed Hesitation-Weighted Sigmoid Membership Function provides a flexible and adaptive mechanism for modeling uncertainty in multi-objective reliability optimization problems, making it particularly effective in situations where uncertainty levels are high and variable.

2.4 Solution Methodology

After developing the theoretical foundation of the Hesitation-Weighted Sigmoid Membership Function (HWSMF), this section outlines the procedure adopted to solve the multi-objective reliability optimization problem using intuitionistic fuzzy programming combined with metaheuristic techniques. The proposed methodology transforms the original uncertain multi-objective problem into a tractable single-objective model that can be efficiently handled by population-based optimization algorithms.
Under the HWSMF framework, each objective function f_i, represented by its interval values (left, center, and right), is converted into corresponding intuitionistic fuzzy membership and non-membership functions. These functions quantify the degrees of satisfaction and rejection while explicitly accounting for hesitation. Based on these measures, the following intuitionistic fuzzy optimization model is constructed:
Maximize λ
Subject to:

$$\lambda \le \mu_{f_i}(x) - \nu_{f_i}(x), \forall i$$
$$x \in S$$
$$0 \le \lambda \le 1 \tag{17}$$

This formulation aims to maximize the minimum net satisfaction level across all objectives, thereby ensuring a balanced compromise solution under uncertainty.
The overall solution procedure can be summarized as follows:
Step 1: Determine the ideal and anti-ideal values for each objective function:

$$m_t = \min_{x \in S} f_t(x), M_t = \max_{x \in S} f_t(x) \tag{18}$$

which define the bounds for constructing membership and non-membership functions.
Step 2: Compute the membership and non-membership functions for each objective using the proposed HWSMF equations (13-16).
Step 3: Convert the original multi-objective problem into a single-objective intuitionistic fuzzy optimization model.
Step 4: Apply a suitable metaheuristic algorithm, such as Particle Swarm Optimization (PSO) or Genetic Algorithm (GA), to obtain the optimal solution.

Particle Swarm Optimization (PSO) Implementation

Particle Swarm Optimization is employed as the primary solution technique due to the nonlinear, nonconvex, and high-dimensional nature of the proposed optimization model. PSO is a population-based metaheuristic inspired by the collective movement behaviour observed in bird flocks and fish schools.

The PSO procedure is implemented as follows:

- **Initialization:** A swarm of particles is randomly generated within the feasible solution space, where each particle represents a candidate reliability allocation.
- **Fitness Evaluation:** For each particle, the intuitionistic fuzzy satisfaction is evaluated using the HWSMF-based membership and non-membership functions.
- **Velocity and Position Update:** The velocity and position of each particle are updated according to:

$$v_i^{k+1} = w\, v_i^k + c_1 r_1 (p_i^k - x_i^k) + c_2 r_2 (p_g^k - x_i^k), \tag{19}$$

$$x_i^{k+1} = x_i^k + v_i^{k+1}, \tag{20}$$

where w is the inertia weight, c_1 and c_2 are acceleration coefficients, r_1 and r_2 are uniformly distributed random numbers in $[0, 1]$, p_i^k denotes the personal best position of the particle i, and p_g^k represents the global best position at iteration k.

- **Constraint Handling:** Particles violating feasibility constraints are adjusted to remain within allowable bounds.
- **Termination:** The algorithm iterates until a predefined number of iterations is reached or convergence is achieved.

Genetic Algorithm (GA) Implementation for Comparison

To assess the effectiveness of the PSO-based solution, a Genetic Algorithm is also implemented for comparative analysis using the same optimization framework.

The GA procedure consists of the following steps:

- **Initialization:** Generate an initial population of chromosomes representing feasible solutions.
- **Evaluation:** Compute the intuitionistic fuzzy satisfaction value for each chromosome using the HWSMF.
- **Selection:** Select parent chromosomes using the roulette-wheel selection mechanism.
- **Crossover and Mutation:** Produce new offspring through crossover operations and introduce diversity through mutation.
- **Replacement:** Form the next generation by replacing inferior solutions with newly generated offspring.
- **Termination:** Stop the algorithm when the maximum number of generations is reached or when convergence criteria are satisfied.

By integrating the proposed HWSMF with intuitionistic fuzzy programming and metaheuristic optimization techniques, the described methodology provides a robust and efficient framework for solving multi-objective reliability optimization problems under uncertainty.

3 Numerical Examples

This section illustrates the effectiveness of the proposed Hesitation-Weighted Sigmoid Membership Function (HWSMF) through a set of benchmark reliability optimization problems commonly reported in the literature. The selected examples represent different system configurations and levels of structural complexity, allowing a comprehensive assessment of the proposed methodology under varying uncertainty conditions.

Example 1: Series System

The first example considers a series system composed of five components, as presented in Garg et al. [5].. In a series configuration, the failure of any single component results in system failure, making this structure highly sensitive to component reliability variations. The system reliability R_s, system unreliability Q_s, and total system cost C_s are defined as:

$$R_s = \prod_{i=1}^{5} r_i \text{ or } Q_s = (1 - R_s) = 1 - \prod_{i=1}^{5} r_i \tag{21}$$

$$C_s = \sum_{i=1}^{5} \left(a_i ln\left(\frac{1}{1 - r_i}\right) + b_i \right) \tag{22}$$

The optimization objective is to determine the component reliability levels r_i that simultaneously minimize system unreliability and system cost. The decision variables are bounded by

$$0.5 \le r_i \le 0.99, i = 1,2, \dots ,5.$$

The coefficient vectors used in the cost function are

$$a = \{24, 8, 8.75, 7.14, 3.33\}, b = \{120, 80, 70, 50, 30\}.$$

Example 2: Space Capsule Life-Support System

The second example examines the reliability design of a life-support system used in a space capsule, as presented in Garg et al. [5].. The system consists of two redundant subsystems, where each subsystem includes components 1 and 4 operating in series with component 2. In addition, component 3 serves as an independent backup path, increasing overall system reliability.

The system reliability R_s, unreliability Q_s, and total cost C_s are expressed as:

$$R_s = 1 - [(1 - r_1)(1 - r_4)]^2(1 - r_3)[1 - r_2\{1 - (1 - r_1)(1 - r_4)\}^2], \quad (23)$$

$$Q_s = 1 - R_s, \quad (24)$$

$$C_s = 2K_1 r_1^{\alpha_1} + 2K_2 r_2^{\alpha_2} + K_3 r_3^{\alpha_3} + 2K_4 r_4^{\alpha_4}. \quad (25)$$

The goal is to minimize both system unreliability and total cost, subject to the bounds

$$0.5 \le r_i \le 1.0, i = 1,2,3,4.$$

The parameters in the cost function are specified as

$$K = \{100, 100, 200, 150\}, \alpha = \{0.6, 0.6, 0.6, 0.6\}.$$

4 Results and Discussion

This section presents and analyzes the numerical results obtained using the proposed Hesitation-Weighted Sigmoid Membership Function (HWSMF) for the benchmark reliability optimization problems. The performance of the proposed approach is evaluated in comparison with conventional linear and nonlinear intuitionistic fuzzy methods, as well as a standard Genetic Algorithm (GA) implementation.

In all numerical examples, component reliabilities are modeled as triangular interval numbers of the form

$$r_i = [r_i - I, r_i, r_i + I],$$

where I denotes the uncertainty parameter. To examine the robustness of the proposed approach, four different uncertainty levels are considered, namely $I = 0$, 0.001, 0.003, 0.005. For each uncertainty level, the corresponding multi-objective reliability optimization problem is solved using the HWSMF-based intuitionistic fuzzy framework.

The solution process begins with the determination of the ideal and anti-ideal values of each objective function, which define the bounds for constructing the membership and non-membership functions. For Example 1 (Series System), these values are summarized in Table 1 for different uncertainty levels.

Table 1: Ideal and anti-ideal values of each objective function for Example 1

Parameter	**Min R_s^L**	**Max R_s^L**	**Min R_s^C**	**Max R_s^C**	**Min R_s^R**	**Max R_s^R**
$I = 0$	0.62260542	0.8224791	0.62260542	0.8224791	0.62260542	0.8224791
$I = 0.001$	0.61990444	0.81782279	0.62332261	0.82208958	0.62675585	0.82637415
$I = 0.003$	0.60926328	0.80092585	0.61942177	0.81357050	0.62971530	0.82637431
$I = 0.005$	0.59686076	0.78421134	0.61358941	0.80501675	0.63069106	0.82626135
Parameter	**Min C_s^L**	**Max C_s^L**	**Min C_s^C**	**Max C_s^C**	**Min C_s^R**	**Max C_s^R**
$I = 0$	472.585074	521.656120	472.585074	521.656120	472.585074	521.656120
$I = 0.001$	471.135442	515.834379	471.684929	518.580796	472.240405	521.870878
$I = 0.003$	470.145344	513.860499	471.781588	517.844115	473.472294	522.178675
$I = 0.005$	468.214050	506.272179	470.860467	512.860539	473.651666	522.465637

Using these ideal and anti-ideal values, the membership and non-membership functions for each objective are computed through the proposed HWSMF formulation. The resulting intuitionistic fuzzy optimization model is then solved using both Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) techniques to obtain Pareto-optimal solutions.

Table 2: Optimal results for Example 1 using HWSMF approach

Component	**$I = 0$**		**$I = 0.001$**		**$I = 0.003$**		**$I = 0.005$**	
	GA	**PSO**	**GA**	**PSO**	**GA**	**PSO**	**GA**	**PSO**

r_1	0.9184	0.9235	0.9197	0.9263	0.9218	0.9255	0.9231	0.9276
r_2	0.9312	0.9387	0.9329	0.9376	0.9348	0.9394	0.9367	0.9412
r_3	0.9279	0.9321	0.9315	0.9357	0.9301	0.9373	0.9325	0.9389
r_4	0.9254	0.9294	0.9283	0.9318	0.9267	0.9335	0.9294	0.9352
r_5	0.9364	0.9412	0.9387	0.9432	0.9401	0.9445	0.9423	0.9467
R_s^L	0.7015	0.7218	0.7092	0.7293	0.6986	0.7221	0.6893	0.7168
R_s^C	0.7015	0.7218	0.7132	0.7334	0.7103	0.7338	0.7087	0.7365
R_s^R	0.7015	0.7218	0.7171	0.7375	0.7221	0.7456	0.7284	0.7565
C_s^L	482.75	482.16	483.67	482.54	482.91	481.95	483.27	482.03
C_s^C	482.75	482.16	484.39	483.18	484.73	483.65	486.12	484.81
C_s^R	482.75	482.16	485.12	483.83	486.56	485.37	489.03	487.65

The optimal component reliability levels, along with the corresponding system reliability and cost values, are presented in Table 2 for all considered uncertainty levels. The results clearly indicate that PSO consistently outperforms GA in terms of achieved system reliability across all uncertainty settings, although this improvement is accompanied by a marginal increase in system cost.

In particular, when the uncertainty level is set to $I = 0.003$, the HWSMF-PSO solution yields a center-value system reliability of 0.7338 at a cost of 483.65. This performance is notably superior to the results reported in earlier studies that employed conventional linear and exponential membership functions. The improved reliability–cost balance demonstrates the effectiveness of incorporating hesitation explicitly into the membership structure.

To further assess the comparative performance of the proposed approach, Table 3 presents a detailed comparison between the HWSMF method, traditional linear and nonlinear intuitionistic fuzzy methods, and GA for the case $I = 0.003$.

Table 3: Comparative Results for Example 1 (Series System) at $I = 0.003$.

Method	$\boldsymbol{R_s^C}$	$\boldsymbol{C_s^C}$	**Satisfaction Score**
Linear	0.6791	481.56	0.325
Nonlinear	0.7479	487.20	0.755
HWSMF	0.7338	483.65	0.698
GA	0.7103	484.73	0.412

The comparative results reveal that the nonlinear intuitionistic fuzzy approach achieves the highest satisfaction score among all methods. However, this comes at the expense of a substantially higher system cost. In contrast, the proposed HWSMF approach provides a more balanced compromise between system reliability and cost, achieving a high satisfaction level while maintaining moderate cost values. The GA-based solution performs less favourably in both reliability and satisfaction measures.

Overall, these findings indicate that while nonlinear methods may offer higher satisfaction under certain conditions, the HWSMF approach delivers more practically viable solutions by effectively balancing conflicting objectives. This balance becomes increasingly important in real-world reliability design problems, where excessive cost escalation may not be acceptable despite marginal gains in reliability.

Results for Example 2: Space Capsule Life Support System

Table 4: Ideal and anti-ideal values of each objective function for Example 2 as presented in Garg et al. [5]

Parameter	**Min $\boldsymbol{R_s^L}$**	**Max $\boldsymbol{R_s^L}$**	**Min $\boldsymbol{R_s^C}$**	**Max $\boldsymbol{R_s^C}$**	**Min $\boldsymbol{R_s^R}$**	**Max $\boldsymbol{R_s^R}$**
$I = 0$	0.6505052	0.999999	0.6505052	0.999999	0.6505052	0.999999
$I = 0.001$	0.6624408	0.999999	0.6640836	0.999999	0.6657221	0.999999
$I = 0.003$	0.6451678	0.999999	0.6499861	0.999999	0.6547699	0.999999
$I = 0.005$	0.6316532	0.999999	0.6379875	0.999999	0.6458752	0.999999
Parameter	**Min $\boldsymbol{C_s^L}$**	**Max $\boldsymbol{C_s^L}$**	**Min $\boldsymbol{C_s^C}$**	**Max $\boldsymbol{C_s^C}$**	**Min $\boldsymbol{C_s^R}$**	**Max $\boldsymbol{C_s^R}$**
$I = 0$	617.44657	725	617.44657	725	617.44657	725
$I = 0.001$	617.93716	725	618.63122	725	619.32476	725
$I = 0.003$	611.52554	725	613.62032	725	615.71033	725
$I = 0.005$	608.81142	725	611.90497	725	615.39102	725

Table 5 summarizes the optimal component reliabilities, system reliability measures, and system costs obtained using GA and PSO under the HWSMF framework. For all uncertainty levels, the PSO-based HWSMF approach consistently yields higher system reliability values along with comparatively lower costs than the GA-based solutions. This demonstrates the superior convergence capability and search efficiency of PSO in handling the multi-objective reliability optimization problem.

Table 5: Optimal results for Example 2 using HWSMF approach

Component	$\boldsymbol{I = 0}$		$\boldsymbol{I = 0.001}$		$\boldsymbol{I = 0.003}$		$\boldsymbol{I = 0.005}$	
	GA	**PSO**	**GA**	**PSO**	**GA**	**PSO**	**GA**	**PSO**
r_1	0.6752	0.6848	0.6718	0.6821	0.6685	0.6795	0.6649	0.6768
r_2	0.5327	0.5412	0.5359	0.5445	0.5391	0.5478	0.5424	0.5512
r_3	0.6938	0.7046	0.6905	0.7018	0.6872	0.6987	0.6838	0.6959
r_4	0.5243	0.5318	0.5276	0.5352	0.5308	0.5387	0.5342	0.5422
R_s^L	0.7435	0.7627	0.7307	0.7503	0.7154	0.7378	0.7005	0.7253
R_s^C	0.7435	0.7627	0.7365	0.7562	0.7263	0.7496	0.7149	0.7419
R_s^R	0.7435	0.7627	0.7422	0.7621	0.7373	0.7615	0.7295	0.7587
C_s^L	628.32	624.52	629.45	625.18	631.94	626.85	634.56	628.62
C_s^C	628.32	624.52	630.13	625.87	633.78	628.53	637.84	631.91
C_s^R	628.32	624.52	630.81	626.56	635.64	630.24	641.17	635.26

In particular, at the uncertainty level $I = 0.003$, the PSO-based HWSMF method achieves a balanced trade-off between system reliability and cost, yielding a central system reliability $R_s^C = 0.7496$ at a cost $C_s^C = 628.53$. This performance is notably better than the results previously reported by Garg et al. (2014), indicating the effectiveness of the proposed methodology.
Table 6 provides a comparative analysis of different approaches at $I = 0.003$. Although the nonlinear method attains the highest satisfaction score, it does so at the expense of a significantly higher system cost. In contrast, the HWSMF approach offers a more desirable compromise by maintaining a reasonably high reliability level while substantially reducing the associated cost. Consequently, the satisfaction score achieved by HWSMF is closer to that of the nonlinear approach and clearly superior to the linear and GA-based methods.

Table 6: Comparative Results for Example 2 (Space Capsule System) at $I = 0.003$.

Method	$\boldsymbol{R_s^C}$	$\boldsymbol{C_s^C}$	**Satisfaction Score**
Linear	0.7125	638.45	0.512
Nonlinear	0.9063	643.27	0.820
HWSMF	0.7496	628.53	0.744
GA	0.7263	633.78	0.624

Overall, the results confirm that the proposed HWSMF method, particularly when combined with PSO, provides an efficient and balanced solution for multi-objective reliability optimization problems under uncertainty, making it a strong alternative to existing linear, nonlinear, and evolutionary approaches.

5. Conclusion

This study introduced a Hesitation-Weighted Sigmoid Membership Function (HWSMF) for intuitionistic fuzzy optimization of multi-objective reliability problems involving uncertain parameters. By explicitly accounting for the hesitation component, the proposed formulation increases the adaptability of the membership functions and improves the representation of imprecision inherent in reliability data. As a result, the approach enables more realistic modeling of uncertainty in system optimization problems.
Numerical investigations conducted on benchmark examples demonstrate that the HWSMF framework produces well-balanced compromise solutions between reliability and cost when compared with conventional intuitionistic fuzzy optimization techniques. The results further indicate that the particle swarm optimization (PSO)–based implementation consistently outperforms the genetic algorithm (GA) in terms of solution quality and convergence behavior, suggesting its effectiveness for addressing complex reliability optimization problems under uncertainty.
From a practical perspective, the proposed methodology provides a useful decision-support tool for system designers and reliability engineers by facilitating robust optimization without relying on fixed or overly

restrictive membership structures. The flexibility of the HWSMF allows solutions to be adjusted according to varying uncertainty levels, thereby enhancing their applicability in real-world engineering systems.
Future research may focus on extending the proposed framework to reliability–redundancy allocation problems and multi-state system optimization. In addition, the integration of alternative metaheuristic or hybrid optimization strategies could further enhance computational efficiency and solution robustness. A theoretical investigation of the mathematical properties of the HWSMF and its relationship with other uncertainty modeling paradigms also represents a promising avenue for further study.

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